TEXTBOOKS IN MATHEMATICS

# MATHEMATICAL MODELING FOR BUSINESS ANALYTICS

William P. Fox

CRC Press
Taylor & Francis Group
A CHAPMAN & HALL BOOK

# Mathematical Modeling for Business Analytics

# TEXTBOOKS in MATHEMATICS

## Series Editors: Al Boggess and Ken Rosen

PUBLISHED TITLES

ABSTRACT ALGEBRA: A GENTLE INTRODUCTION
Gary L. Mullen and James A. Sellers

ABSTRACT ALGEBRA: AN INTERACTIVE APPROACH, SECOND EDITION
William Paulsen

ABSTRACT ALGEBRA: AN INQUIRY-BASED APPROACH
Jonathan K. Hodge, Steven Schlicker, and Ted Sundstrom

ADVANCED LINEAR ALGEBRA
Hugo Woerdeman

ADVANCED LINEAR ALGEBRA
Nicholas Loehr

ADVANCED LINEAR ALGEBRA, SECOND EDITION
Bruce Cooperstein

APPLIED ABSTRACT ALGEBRA WITH MAPLE™ AND MATLAB®, THIRD EDITION
Richard Klima, Neil Sigmon, and Ernest Stitzinger

APPLIED DIFFERENTIAL EQUATIONS: THE PRIMARY COURSE
Vladimir Dobrushkin

APPLIED DIFFERENTIAL EQUATIONS WITH BOUNDARY VALUE PROBLEMS
Vladimir Dobrushkin

APPLIED FUNCTIONAL ANALYSIS, THIRD EDITION
J. Tinsley Oden and Leszek Demkowicz

A BRIDGE TO HIGHER MATHEMATICS
Valentin Deaconu and Donald C. Pfaff

COMPUTATIONAL MATHEMATICS: MODELS, METHODS, AND ANALYSIS WITH MATLAB® AND MPI, SECOND EDITION
Robert E. White

A CONCRETE INTRODUCTION TO REAL ANALYSIS, SECOND EDITION
Robert Carlson

A COURSE IN DIFFERENTIAL EQUATIONS WITH BOUNDARY VALUE PROBLEMS, SECOND EDITION
Stephen A. Wirkus, Randall J. Swift, and Ryan Szypowski

A COURSE IN ORDINARY DIFFERENTIAL EQUATIONS, SECOND EDITION
Stephen A. Wirkus and Randall J. Swift

DIFFERENTIAL EQUATIONS: THEORY, TECHNIQUE, AND PRACTICE, SECOND EDITION
Steven G. Krantz

DIFFERENTIAL EQUATIONS: THEORY, TECHNIQUE, AND PRACTICE WITH BOUNDARY VALUE PROBLEMS
Steven G. Krantz

DIFFERENTIAL EQUATIONS WITH APPLICATIONS AND HISTORICAL NOTES, THIRD EDITION
George F. Simmons

DIFFERENTIAL EQUATIONS WITH MATLAB®: EXPLORATION, APPLICATIONS, AND THEORY
Mark A. McKibben and Micah D. Webster

DISCOVERING GROUP THEORY: A TRANSITION TO ADVANCED MATHEMATICS
Tony Barnard and Hugh Neill

DISCRETE MATHEMATICS, SECOND EDITION
Kevin Ferland

ELEMENTARY DIFFERENTIAL EQUATIONS
Kenneth Kuttler

ELEMENTARY NUMBER THEORY
James S. Kraft and Lawrence C. Washington

THE ELEMENTS OF ADVANCED MATHEMATICS: FOURTH EDITION
Steven G. Krantz

ESSENTIALS OF MATHEMATICAL THINKING
Steven G. Krantz

EXPLORING CALCULUS: LABS AND PROJECTS WITH MATHEMATICA®
Crista Arangala and Karen A. Yokley

EXPLORING GEOMETRY, SECOND EDITION
Michael Hvidsten

EXPLORING LINEAR ALGEBRA: LABS AND PROJECTS WITH MATHEMATICA®
Crista Arangala

EXPLORING THE INFINITE: AN INTRODUCTION TO PROOF AND ANALYSIS
Jennifer Brooks

GRAPHS & DIGRAPHS, SIXTH EDITION
Gary Chartrand, Linda Lesniak, and Ping Zhang

INTRODUCTION TO ABSTRACT ALGEBRA, SECOND EDITION
Jonathan D. H. Smith

INTRODUCTION TO ANALYSIS
Corey M. Dunn

PUBLISHED TITLES CONTINUED

INTRODUCTION TO MATHEMATICAL PROOFS: A TRANSITION TO ADVANCED MATHEMATICS, SECOND EDITION
Charles E. Roberts, Jr.

INTRODUCTION TO NUMBER THEORY, SECOND EDITION
Marty Erickson, Anthony Vazzana, and David Garth

INVITATION TO LINEAR ALGEBRA
David C. Mello

LINEAR ALGEBRA, GEOMETRY AND TRANSFORMATION
Bruce Solomon

MATHEMATICAL MODELLING WITH CASE STUDIES: USING MAPLE™ AND MATLAB®, THIRD EDITION
B. Barnes and G. R. Fulford

MATHEMATICS IN GAMES, SPORTS, AND GAMBLING–THE GAMES PEOPLE PLAY, SECOND EDITION
Ronald J. Gould

THE MATHEMATICS OF GAMES: AN INTRODUCTION TO PROBABILITY
David G. Taylor

A MATLAB® COMPANION TO COMPLEX VARIABLES
A. David Wunsch

MEASURE AND INTEGRAL: AN INTRODUCTION TO REAL ANALYSIS, SECOND EDITION
Richard L. Wheeden

MEASURE THEORY AND FINE PROPERTIES OF FUNCTIONS, REVISED EDITION
Lawrence C. Evans and Ronald F. Gariepy

NUMERICAL ANALYSIS FOR ENGINEERS: METHODS AND APPLICATIONS, SECOND EDITION
Bilal Ayyub and Richard H. McCuen

ORDINARY DIFFERENTIAL EQUATIONS: AN INTRODUCTION TO THE FUNDAMENTALS
Kenneth B. Howell

PRINCIPLES OF FOURIER ANALYSIS, SECOND EDITION
Kenneth B. Howell

REAL ANALYSIS AND FOUNDATIONS, FOURTH EDITION
Steven G. Krantz

RISK ANALYSIS IN ENGINEERING AND ECONOMICS, SECOND EDITION
Bilal M. Ayyub

SPORTS MATH: AN INTRODUCTORY COURSE IN THE MATHEMATICS OF SPORTS SCIENCE AND SPORTS ANALYTICS
Roland B. Minton

PUBLISHED TITLES CONTINUED

A TOUR THROUGH GRAPH THEORY
Karin R. Saoub

TRANSITION TO ANALYSIS WITH PROOF
Steven G. Krantz

TRANSFORMATIONAL PLANE GEOMETRY
Ronald N. Umble and Zhigang Han

Taylor & Francis
Taylor & Francis Group
http://taylorandfrancis.com

# Mathematical Modeling for Business Analytics

by

William P. Fox

CRC Press
Taylor & Francis Group
Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742



Printed on acid-free paper

International Standard Book Number-13: 978-1-1385-5661-4 (Hardback)



**Library of Congress Cataloging-in-Publication Data**

Names: Fox, William P., 1949- author.
Title: Mathematical modeling for business analytics / William P. Fox.
Description: Boca Raton, FL : CRC Press, 2018.
Identifiers: LCCN 2017034022 | ISBN 9781138556614
Subjects: LCSH: Decision making--Mathematical models. |
Management--Mathematical models.
Classification: LCC HD30.23 .F7325 2018 | DDC 658.4/033--dc23
LC record available at https://lccn.loc.gov/2017034022

**Visit the Taylor & Francis Web site at**
**http://www.taylorandfrancis.com**

**and the CRC Press Web site at**
**http://www.crcpress.com**

*This book is dedicated to my wife, Hamilton Dix-Fox,*

*who encouraged me to write this book.*

# Contents

# Preface

## Addressing the Current Needs

In recent years of teaching mathematical modeling for decision-making coupled with conducting applied mathematical modeling research, I have found that (a) decision-makers at all levels must be exposed to the tools and techniques that are available to help them in the decision-making process, (b) decision-makers and analysts need to have and use technology to assist in the analysis process, and (c) the interpretation and explanation of the results are crucial to understanding the strengths and limitations of modeling. With this in mind this book emphasizes and focuses on the model formulation and modeling building skills that are required for decision analysis and the technology to support the analysis.

## Audience

This book will be best used for a senior-level discrete modeling course in mathematics, operations research, or industrial engineering departments or graduate-level discrete choice modeling courses, or decision models courses offered in business schools offering business analytics. The book *would be* of interest to mathematics departments that offer mathematical modeling courses focused on discrete modeling.

The following groups would benefit from using this book:

- Undergraduate students who are involved in quantitative methods courses in business, operations research, industrial engineering, management sciences, industrial engineering, or applied mathematics.
- Graduate students in discrete mathematical modeling courses covering topics from business, operations research, industrial engineering, management sciences, industrial engineering, or applied mathematics.
- Junior analysts who want a comprehensive review of decision-making topics.
- Practitioners desiring a reference book.

## Objective

The primary objective of this book is illustrative in nature. It seeks the tone in Chapter 1 through the introduction to mathematical modeling. In this chapter, we provide a process for formally thinking about the problem. We illustrate many scenarios and illustrative examples. In these examples, we begin the setup of the solution process. We also provide solutions in this chapter that we will present more in depth in later chapters.

We thought for years about which techniques should be included or excluded in this book. Finally, we decided on the main techniques that we cover in our three-course sequence in mathematical modeling for decision-making in the Department of Defense Analysis. We feel these subjects have served our students as well as they have gone on as leaders and decision-makers for our nation.

## Organization

This book contains information that could easily be covered in a two-semester course or a one-semester overview of topics. This allows instructors the flexibility to pick and choose topics that are consistent with their course and consistent with the current needs.

In Chapters 2 through 8, we present materials to solve the typical problems introduced in Chapter 1. The contexts of these problems are in business, industry, and government (BIG). Thus, the problems visited are BIG problems.

In Chapter 2, we present a decision theory. We discuss decision under uncertainty and risk.

We present the process of using decision trees to draw out the problem and then use the expected value to solve for the *best* decision.

Chapter 3 has developed into a mathematical programming covering techniques and applications in technology to solve linear, integer, and nonlinear optimization problems. Among the problems illustrated are a supply chain operation, a recruiting office analysis, emergency service planning, optimal path to transport hazardous materials, a minimum variance investments, and cable installation.

Chapter 4 covers the techniques to rank entities or alternatives when the decision-maker has criteria to be considered that impact the decision-making process. Discussions and illustrative problems are presented in data envelopment analysis using linear programming (LP), sum of additive weights, analytical hierarchy process (AHP), and technique for order of preference by similarity to ideal solution (TOPSIS). In our current modeling courses, we present these techniques to our students who are finding more and more applications in BIG.

Chapter 5 covers game theory when decisions are made versus an opponent. We assume rational players, simultaneous games with perfect knowledge in our presentation. We do cover both total conflict and partial conflict applications. Applications are presented that covers the following:

Battle of the Bismarck Sea, Penalty kicks in soccer, Batter–Pitcher Duel in baseball, Operation Overlord, Choosing the right Course of Action, Cuban Missile Crisis, 2007–2008 Writer's Guild Strike, Dark Money Network (DMN) Game, and Course of Actions Revisited.

In Chapter 6, we present an overview of regression techniques. We have found in our advising that students (future decision-makers) often use the wrong technique in the work. Here, we present simple linear regression, multiple linear regression, nonlinear regression, logistics regression, and Poisson regression. The approach here is when to use each type of regression based on the data available.

Chapter 7 presents discrete dynamical systems (DDS). We cover not only simple linear models but nonlinear as well as system of systems. We discuss stability and equilibrium values.

Illustrative example include the following: drug dosage, time value of money, simple mortgage, population growth, spread of a contagious disease, inventory systems analysis, competitive hunter model, Lanchester's combat models, discrete predator–prey model, and a systems model for disease known as the susceptible (S), infected (I), and resistant (R) (SIR) model.

Chapter 8 is a brief presentation of discrete simulation models. Often models cannot be constructed to adequately reflect the fidelity of the system of interest. In these cases, simulation models, especially Monte Carlo simulations, provide a look into the system that provides information to the decision-makers.

Chapter 9 presents an introduction into financial mathematics. The topics from engineering economic analysis such as rates of interest, depreciation, discounting, annual percentage rate (APR), compounding (discrete and continuous), net present value (NPV), bonds, annuities, and shrinking funds are presented and explained so that they can be easily understood and used. We also discuss examples using previously covered techniques such as multi-attribute decision-making, dynamical systems, and mathematical programming that are applied directly to financial mathematics. In our research on institutions that provided interest, we found none that gave a continuous interest. They all gave interest at discrete intervals. At our local credit union, their sign says money deposited after 10 a.m. will be credited until after 10 a.m. the following day. So many examples from engineering economic analysis are derived in the discrete model using DDS. Examples for pensions with stock portfolios, financial planning, optimization of interest, and cash flow are discussed using the appropriate mathematical technique.

This book shows the power and limitations for mathematical modeling to solve real problems. The solutions shown might not be the best solution but they are certainly solutions that are or could be considered in the analysis. As evidenced by previous textbooks in mathematical modeling, such as a *First Course in Mathematical Modeling*, scenarios are revisited to illustrate alternative techniques in solving problems. As we have seen from many years of the Mathematical Contest in Modeling, student ingenuity and creativity in modeling methods and solution techniques are always present.

In this book, we cannot address every nuance in modeling real-world problems. What we can do is provide a sample of models and possible appropriate techniques to obtain useful results. We can establish a process to *do modeling*. We can illustrate many examples of modeling and illustrate a technique in order to solve the problem. In the techniques chapters, we must assume no background and spend a little time establishing the procedure before we return to providing examples.

The data used in this book are unclassified and often the real data are not displayed. Data similar to nature and design are used in the examples.

This book can apply to analysts to allow them to see the range and type of problems that fit into specific mathematical techniques understanding that we did address all the possible mathematics techniques. Some important techniques that we left out include differential equations.

This book is also applicable to decision-makers. It shows the decision-maker the wide range of applications of quantitative approaches to aid in the decision-making process. As we say in class every day, mathematics does not tell what to do but it does provide insights and allows critical thinking into the decision-making process. In our discussion, we consider the mathematical modeling process as a framework for decision-makers. For a decision-maker there are four key elements: (1) the formulation process, (2) the solution process, (3) interpretation of the mathematical answer in the context of the actual problem, and (4) sensitivity analysis. At every step along the way in the process the decision-maker should question procedures and techniques and ask for further explanations as well as assumptions used in the process. One major question could be, "Did you use an appropriate technique?" to obtain a solution and "Why were other techniques not considered or used?" Another question could be "Did you over simplify the process?" so much that the solution does not really apply in order or were the assumptions make fundamental to even being able to solve the problem?

We thank all the mathematical modeling students that we have had over this time as well as all the colleagues who have taught mathematical modeling with us during this adventure. I am especially appreciative of the mentorship of Frank R. Giordano over the past thirty-plus years.

**William P. Fox**
*Naval Postgraduate School*

# *Author*

**Dr. William P. Fox** is currently a professor in the Department of Defense Analysis at the Naval Postgraduate School, Monterey, California and teaches a three-course sequence in mathematical modeling for decision-making. He earned his BS degree from the United States Military Academy at West Point, New York, MS in operations research from the Naval Postgraduate School, and PhD in industrial engineering from Clemson University, Clemson, South Carolina. He has a teaching experience of 12 years at the United States Military Academy until retiring for his active military service and he was the chair of mathematics for 8 years at the Francis Marion University, Florence, South Carolina. He has many publications and scholarly activities, including 16 books, 21 book chapters and technical reports, 150 journal articles, and more than 150 conference presentations and mathematical modeling workshops. He has directed several international mathematical modeling contests through the Consortium of Mathematics and Its Applications (COMAP): the HiMCM and the MCM. His interests include applied mathematics, optimization (linear and nonlinear), mathematical modeling, statistical models, models for decision-making in business, industry, medical, and government, and computer simulations. He is a member of the Institute for Operations Research and the Management Sciences (INFORMS), the Military Application Society of INFORMS, the Mathematical Association of America, and the Society for Industrial and Applied mathematics where he has held numerous positions.

Taylor & Francis
Taylor & Francis Group
http://taylorandfrancis.com

# 1

# *Introduction to Mathematical Modeling for Business Analytics*

**OBJECTIVES**

1. Understand the modeling process.
2. Know and use the steps in modeling.
3. Experience a wide variety of examples.

## 1.1 Introduction

Consider the importance of modeling for decision-making in business (B), industry (I), and government (G), BIG. BIG decision-making is essential for success at all levels. We do not encourage *shooting from the hip* or simply flipping a coin to make a decision. We recommend good analysis that enables the decision-maker to examine and question results to find the best alternative to choose or decision to make. This book presents, explains, and illustrates a modeling process and provides examples of decision-making analysis throughout.

Let us describe a mathematical model as a mathematical description of a system by using the language of mathematics. Why mathematical modeling? Mathematical modeling, business analytics, and operations research are all similar descriptions that represent the use of quantitative analysis to solve real problems. This process of developing such a mathematical model is termed as mathematical modeling. Mathematical models are used in the natural sciences (such as physics, biology, earth science, and meteorology), engineering disciplines (e.g., computer science, systems engineering, operations research, and industrial engineering), and in the social sciences (such as business, economics, psychology, sociology, political science, and social networks). The professionals in these areas use mathematical models all the time.

A mathematical model may be used to help explain a system, to study the effects of different components, and to make *predictions* about behavior (Giordano et al., 2014, pp. 58–60). So let us make a more formal definition of a mathematical model: a mathematical model is the application of mathematics to a real-world problem.

Mathematical models can take many forms, including but not limited to dynamical systems: statistical models, differential equations, optimization models, or game theoretic models. These and other types of models can overlap, of which one output becomes the input for another similar or different model forms. In many cases, the quality of a scientific field depends on how well the mathematical models developed on the theoretical side agree with the results of repeatable experiments (Giordano et al., 2014, pp. 58–60). Any lack of agreement between theoretical mathematical models and experimental measurements leads to model refinements and better models. We do not plan to cover all the mathematical modeling processes here. We only provide an overview to the decision-makers. Our goal is to offer *competent*, *confident problem solver*s for the twenty-first century. We suggest the books listed in the reference section to become familiar with many more modeling forms.

## 1.2 Background

### 1.2.1 Overview and Process of Mathematical Modeling

Bender (2000, pp. 1–8) first introduced a process for modeling. He highlighted the following: formulate the model, outline the model, ask if it is useful, and test the model. Others have expanded this simple outlined process. Giordano et al. (2014, p. 64) presented a six-step process: identify the problem to be solved, make assumptions, solve the model, verify the model, implement the model, and maintain the model. Myer (2004, pp. 13–15) suggested some guidelines for modeling, including formulation, mathematical manipulation, and evaluation. Meerschaert (1999) developed a five-step process: ask the question, select the modeling approach, formulate the model, solve the model, and answer the question. Albright (2010) subscribed mostly to concepts and process described in previous editions of Giordano et al. (2014). Fox (2012, pp. 21–22) suggested an eight-step approach: understand the problem or question, make simplifying assumptions, define all variables, construct the model, solve and interpret the model, verify the model, consider the model's strengths and weaknesses, and implement the model.

Most of these pioneers in modeling have suggested similar starts in understanding the problem or question to be answered and in making key assumptions to help enable the model to be built. We add the need for sensitivity analysis and model testing in this process to help ensure that we have a model that is performing correctly to answer the appropriate questions.

For example, student teams in the Mathematical Contest in Modeling were building models to determine the all-time best college sports coach. One team picked a coach who coached less than a year, went undefeated for the remaining part of the year, and won their bowl game. Thus, his season was a perfect season. Their algorithm picked this person as the all-time best coach. Sensitivity analysis and model testing could have shown the fallacy to their model.

Someplace between the defining of the variables and the assumptions, we begin to consider the model's form and technique that might be used to solve the model. The list of techniques is boundless in mathematics, and we will not list them here. Suffice it to say that it might be good to initially decide among the forms: deterministic or stochastic for the model, linear or nonlinear for the relationship of the variables, and continuous or discrete.

For example, consider the following scenarios:

Two observation posts that are 5.43 miles apart pick up a brief radio signal. The sensing devices were oriented at 110° and 119°, respectively, when a signal was detected. The devices are accurate to within 2° (that is ±2° of their respective angle of orientation). According to intelligence, the reading of the signal came from a region of active terrorist exchange, and it is inferred that there is a boat waiting for someone to pick up the terrorists. It is dusk, the weather is calm, and there are no currents. A small helicopter leaves a pad from Post 1 and is able to fly accurately along the 110° angle direction. This helicopter has only one detection device, a searchlight. At 200 ft, it can just illuminate a circular region with a radius of 25 ft. The helicopter can fly 225 miles in support of this mission due to its fuel capacity. Where do we search for the boat? How many search helicopters should you use to have a *good* chance of finding the target (Fox and Jaye, 2011, pp. 82–93)?

The writers of TV and movies decide that they are not receiving fair compensation for their work as their shows continue to be played on cable and DVDs. The writers decide to strike. Management refuses to budge. Can we analyze this to prevent this from reoccurring? Can we build a model to examine this?

Consider locating emergency response teams within a county or region. Can we model the location of ambulances to ensure that the maximum number of potential patients is covered by the emergency response teams? Can we find the minimum number of ambulances required?

You are a new manager of a bank. You set new goals for your tenure as manager. You analyze the current status of service to measure against your goals. Are you meeting demand? If not what can be done to improve service? You want to prevent catastrophic failure at your bank.

> You have many alternatives to choose from for your venture. You have certain decision criteria that you consider to sue to help in making this future. Can we build a mathematical model to assist us in this decision?

These are all events that we can model using mathematics. This chapter will help a decision-maker understand what a mathematical modeler might do for them as a confident problem solver using the techniques of mathematical modeling. As a decision-maker, understanding the possibilities and asking the key questions will enable better decisions to be made and will lower the risks.

### 1.2.2 The Modeling Process

We introduce the process of modeling and examine many different scenarios in which mathematical modeling can play a role.

The art of mathematical modeling is learned through experience of building and solving models. Modelers must be creative, innovative, inquisitive, and willing to try new techniques as well as being able to refine their models, if necessary. A major step in the process is *passing the common sense* test for use of the model.

In its basic form, modeling consists of three steps:

1. Make assumptions
2. Do some *math*
3. Derive and interpret conclusions

To that end, one cannot question the mathematics and its solution, but one can always question the assumptions used.

To gain insight, we will consider one framework that will enable the modeler to address the largest number of problems. The key is that there is something *changing for which we want to know the effects and the results of the effects*. The problem might involve any system under analysis. The real-world system can be very simplistic or very complicated. This requires both types of real-world systems to be modeled with the same logical step-wise process.

Consider modeling an investment. Our first inclination is to use the equations about compound interest rates that we used in high school or college algebra. The compound interest formula calculates the value of a compound interest investment after "$n$" interest periods.

$$A = P(1-i)^n$$

where:

$A$ is the amount after $n$ interest periods
$P$ is the principal, the amount invested at the start
$i$ is the interest rate applying to each period
$n$ is the number of interest periods

This is a continuous formula. Have you seen any banking institutions that give continuous interest? In our research, we have not. As a matter of fact at our local credit union, they have a sign that says, money deposited after 10 a.m. do not get credited until the night after the deposit. This makes discrete compound interest on the balance in a more compelling assumption.

A powerful paradigm that we use to model with discrete dynamical systems is as follows:

$$\text{Future value} = \text{present value} + \text{change}$$

The dynamical systems that we will study with this paradigm may differ in appearance and composition, but we will be able to solve a large class of these *seemingly* different dynamical systems with similar methods. In this chapter, we will use iteration and graphical methods to answer questions about the discrete dynamical systems.

We could use flow diagrams to help us see how the dependent variable changes. These flow diagrams help to see the paradigm and put it into mathematical terms. Let us consider financing a new Ford Mustang. The cost is $25,000 and you can put down $2,000, so you need to finance $23,000. The dealership offers you 2% financing over 72 months. Consider the change diagram, Figure 1.1, for financing the car that depicts this situation.

We use this change diagram to help build the discrete dynamical model. Let $A(n)$ be the amount owed after $n$ months. Notice that the arrow pointing into the circle is the interest to the unpaid balance that increases your debt. The arrow pointing out of the circle is your monthly payment that decreases your debt. We define the following variables:

$A(n + 1)$ is the amount owed in the future
$A(n)$ is the amount currently owed

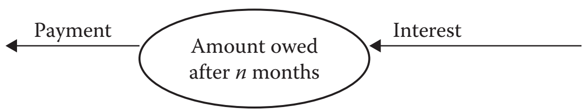


**FIGURE 1.1**
Flow diagram for buying a new car.

Change as depicted in the change diagram is $i\,A(n) - P$, so the model is

$$A(n+1) = A(n) + i\,A(n) - P$$

where:
- $i$ is the monthly interest rate
- $P$ is the monthly payment

We solve by iterating this equation in a spreadsheet. First, we assume that we deposit $1000 at 2.5% annual interest.

We will use the notation $A(n)$ to be the amount of money that we have in our account after $n$ periods. Using the paradigm we have the initial model as

$$A(n+1) = A(n) + \text{change}$$

We model change with the interest rate information and how it is compounded. A change diagram is suggested for more complicated dynamical system models (Figure 1.2).

$$A(0) = \$1000$$

After 1 month, we have

$$1000 + (1000 * 0.025 / 12) = \$1002.08$$

If we want to see after 10 years or 120 months, we are better off using technology such as Excel. We will have $1281.02. We did not fare well, so perhaps we would seek other ways to invest and make this money grow.

Consider installing cable to link computers together in a new computer room. Our first inclination is to use the equations about distance that we used in high school mathematics class. These equations are very simplistic and ignore many factors that could impact the installation of wire lines such as location of terminals, ground or ceiling, and other factors. As we add more factors, we can improve the precision of the model. Adding these additional factors makes the model more realistic but possibly more complicated to produce and solve.

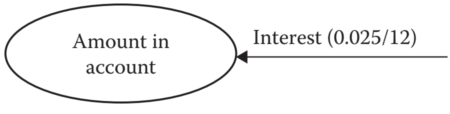


**FIGURE 1.2**
Change diagram for a savings account.

Figure 1.1 provides a closed loop process for modeling. Given a real-world situation similar to the one that is mentioned earlier, we collect data to formulate a mathematical model. This mathematical model can be one that we derive or select from a collection of already existing mathematical models. Then, we analyze the model that we used and reach mathematical conclusions about it. Next, we interpret the model and either makes predict about what has occurred or offer explanation as to why something has occurred. Finally, we test our conclusion about the real-world system with new data. We use sensitivity analysis of the parameters or inputs to see how they affect the model. We may refine or improve the model to improve its ability to predict or explain the phenomena. We might even go back and reformulate a new mathematical model.

### 1.2.3 Mathematical Modeling for Business Analytics as a Process

We will illustrate some mathematical models that describe change in the real world. We will solve some of these models and will analyze how good our resulting mathematical explanations and predictions are in context of the problem. The solution techniques that we employ take advantage of certain characteristics that the various models enjoy as realized through the formulation of the model.

When we observe change, we are often interested in understanding or explaining why or how a particular change occurs. Maybe we need or want to analyze the effects under different conditions or perhaps to predict what could happen in the future. Consider the firing of a weapon system or the shooting of a ball from a catapult as shown in Figure 1.3. Understanding how the system behaves in different environments under differing weather or operators, or predicting how well it hits the targets are all of interest. For the catapult, the critical elements of the ball, the tension, and angle of the firing arm are found as important elements (Fox, 2013b). For our purposes, we will consider a mathematical model to be a mathematical construct that

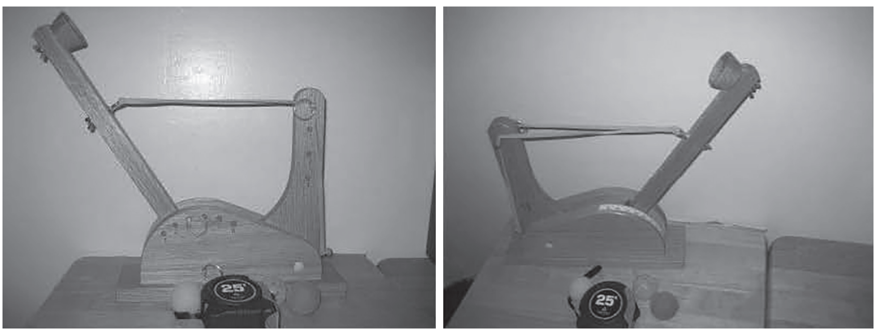

**FIGURE 1.3**
The catapult and balls.

is designed to study a particular real-world system or behavior (Giordano et al., 2014). The model allows us to use mathematical operations to reach mathematical conclusions about the model as illustrated in Figure 1.4. It is the arrow going from real-world system and observations to the mathematical model using the assumptions, variables, and formulations that are critical in the process.

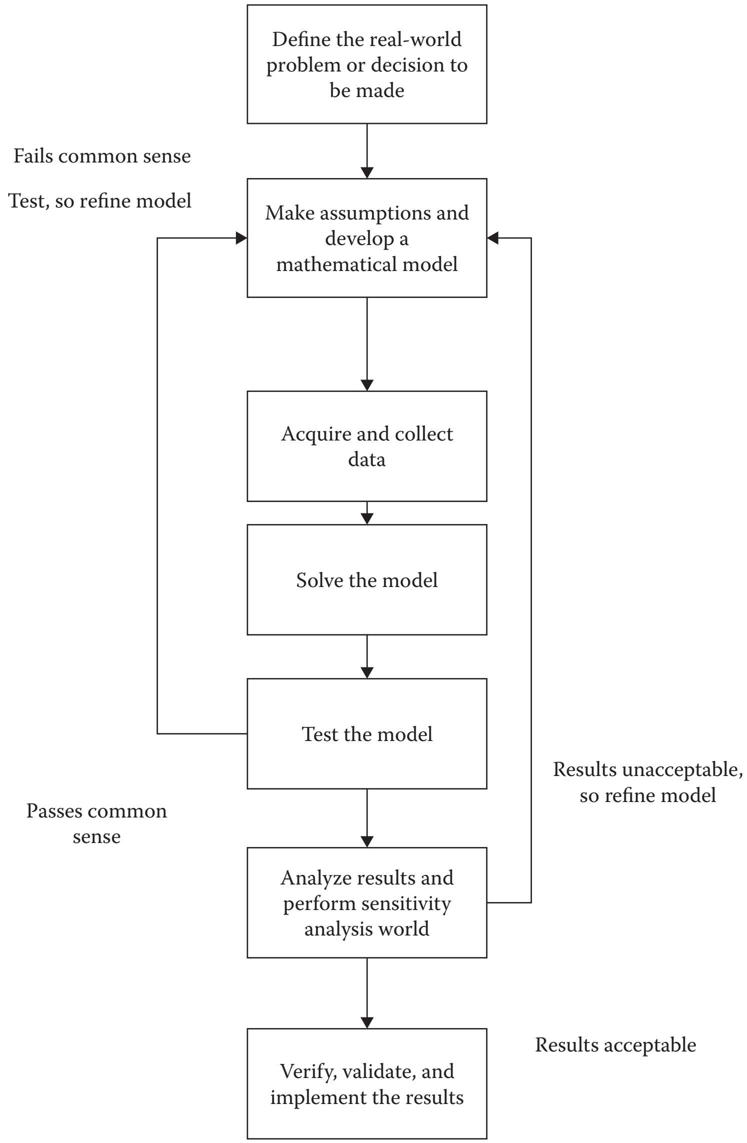


**FIGURE 1.4**
Modeling real-world systems with mathematics.

We define a system as a set of objects joined by some regular interaction or interdependence in order for the complete system to work together. Think of a larger business with many companies that work independently and interact together to make the business prosper. Other examples might include a bass and trout population living in a lake, a communication, cable TV, or weather satellite orbiting the earth, delivering Amazon Prime packages, U.S. postal service mail or packages, locations of emergency services or computer terminals, or large companies' online customer buying systems. The person modeling is interested in understanding how a system works, what causes change in a system, and the sensitivity of the system to change. Understanding all these elements will help in building an adequate model to replicate reality. The person modeling is also interested in predicting what changes might occur and when these changes might occur.

Figure 1.5 suggests how we can obtain real-world conclusions from a mathematical model. First, observations identify the factors that seem to be involved in the behavior of interest. Often we cannot consider, or even identify, all the relevant factors, so we make simplifying assumptions excluding some of those factors (Giordano et al., 2014). Next, we determine what data are available and what variables they represent. We might build or test tentative relationships among the remaining identified factors. For example, if we were modeling the weight of a fish in a fish contest and if we were collecting data on the length and girth of a fish, perhaps we would want to test the relationship of length and girth before we used both variables. This might give us a reasonable first cut at a model. We then solve the model and determine the reasonableness of the model's conclusions. Passing the *common sense* test is important. However, as these results apply only to the model, they may or may not apply to the actual real-world system in question. Simplifications

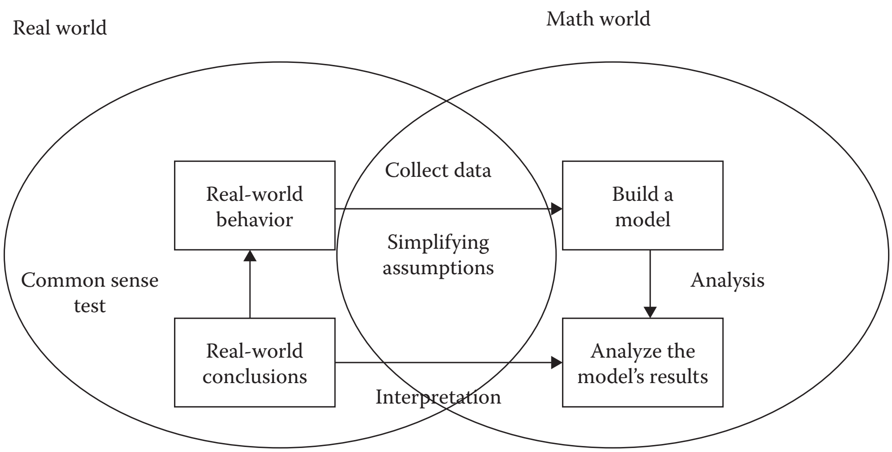


**FIGURE 1.5**
A closed system for modeling.

were made in constructing the model, and the observations on which the model is based invariably contain errors and limitations. Thus, we must carefully account for these anomalies and must test the conclusions of the model against real-world observations. If the model is reasonably valid, we can then draw inferences about the real-world behavior from the conclusions drawn from the model. In summary, the critical elements are the modeling assumptions and variables. The mathematical model used is usually not questionable but why we used it might be questionable. Therefore, we have the following procedure for investigating real-world behavior and for building a mathematical representation:

1. Observe the system and identify the factors and variables involved in the real-world behavior, possibly making simplifying assumptions as necessary.
2. Build initial relationships among the factors and variables.
3. Build the model and analyze the model's results.
4. Interpret the mathematical results both mathematically and in terms of the real-world system.
5. Test the model results and conclusions against real-world observations. Do the results and use of the model pass the common sense test? If not go back and remodel the system.

There are various kinds of models. A good mathematical modeler will build a library of models to recognize various real-world situations to which they apply. Most models simplify reality. Generally, models can only approximate real-world behavior. Next, let us summarize a multistep *process* for formulating a mathematical model.

### 1.2.4 Steps in Model Construction

An outline is presented as a procedure to help construct mathematical models. In the next section, we will illustrate this procedure with a few examples. We suggest a nine-step process.

These nine steps are summarized in Figure 1.6. These steps act as a guide for thinking about the problem and getting started in the modeling process. We choose these steps from the compilation of steps by other authors listed in additional readings and put them together in these nine steps.

We illustrate the process through an example. Consider building a model where we want to identify the spread of a contagious disease.

*Step 1*: Understand the decision to be made, the question to be asked, or the problem to be solved.

Understanding the decision is the same as identifying the problem to be solved. Identifying the problem to study is usually difficult.

| |
|---|
| Step 1: Understand the decision to be made or the question asked.<br>Step 2: Make simplifying assumptions.<br>Step 3: Define all variables.<br>Step 4: Construct a model.<br>Step 5: Solve and interpret the model. Test the model. Do the results pass the *common sense test*?<br>Step 6: Verify the model. Validate the model, if possible<br>Step 7: Identify the strenghts and weaknesses as a reflection of your model.<br>Step 8: Sensitivity analysis and/or model testing<br>Step 9: Implement and maintain the model for future use if it passes the common sense test |

**FIGURE 1.6**
Mathematical modeling process.

In real life, no one walks up to you and hands you an equation to be solved. Usually, it is a comment like "we need to make more money" or "we need to improve our efficiency." Perhaps, we need to make better decisions or we need all our units that are not 100% efficient to become more efficient. We need to be precise in our formulation of the mathematics to actually describe the situation that we need to solve. In our example, we want to identify the spread of a contagious disease to determine how fast it will spread within our region. Perhaps, we will want to use the model to answer the following questions:

1. How long will it take until one thousand people get the disease?
2. What actions may be taken to slow or eradicate the disease?

*Step 2*: Make simplifying assumptions.

Giordano et al. (2014, pp. 62–65) described this well. Again, we suggest starting by brain storming the situation. Make a list of as many factors, or variables, as you can. Now, we realize that we usually cannot capture all these factors influencing a problem in our initial model. The task now is simplified by reducing the number of factors under consideration. We do this by making simplifying assumptions about the factors, such as holding certain factors as constants or ignoring some in the initial modeling phase. We might then examine to see if relationships exist between the remaining factors (or variables). Assuming simple relationships might reduce the complexity of the problem. Once you have a shorter list of variables, classify them as independent variables, dependent variables, or neither.

In our example, we assume we know the type of disease, how it is spread, the number of susceptible people within our region, and what type of medicine is needed to combat the disease. Perhaps, we assume that we know the size of population and the approximate number susceptible to getting the disease.

*Step 3*: Define all variables.

It is critical to define all your variables and provide the mathematical notation for each. In addition, if your variables have units, include them as well. Include all variables, even those you might think that you will not initially use. Often we find that we need these variables later in the refinement process.

In our example, the variables of interest are the number of people currently infected, the number of people who are susceptible to the disease, and the number of people who recently recovered from the disease.

*Step 4*: Construct a model.

Using tools at your disposal or after learning new mathematical tools, you use your own creativity to build a model that describes the situation and the solution of which helps to answer important questions that have been asked. Generally, three methods might be applied here. From first principles, your assumptions, and your variable list, construct a useable mathematical model. From a data set, perform data analysis to examine useful patterns that might suggest a useful model form such as a regression model. From research, take a model off the shelf and either use it directly or modify it appropriately for your use (a good discussion is found in Giordano et al. 2014).

In our example, we find that we might be able to initially use the susceptible (S), infected (I), and resistant (R) (SIR) model off the shelf.

*Step 5*: Solve and interpret the model.

We take the model that we have constructed in Steps 1–4 and we solve it using mathematical tools. Often this model might be too complex or unwieldy, so we cannot solve it or interpret it. If this happens, we return to Steps 2–4 and simplify the model further. We can always try to enhance the model later. We must also ensure that the model yields useable results for which the model was proposed. We will call this *passing the common sense test*.

In our example, the system of discrete dynamical system SIR equations has no closed form analytical solution. It does have graphical and numerical solutions that can be analyzed.

*Step 6*: Verify the model (Giordano et al., 2014, pp. 63–64).

Before we use the model, we should test it out. There are several questions we must ask. Does the model directly answer the question or does the model allow for the answer to the questions to be answered? Is the model useable in a practical sense (can we obtain data to use the model)? Does the model pass the common sense test?

We provide an example to show this. We used a data set whose plot was reasonably a decreasing linear model. The correlation value was 0.94, which meant that the data were strongly linear. We built the linear model and used it to predict the value of a future $y$, where $y$ had to be a positive value. The value of $y$, for our input $x$, was −23.5. So although many of the diagnostics were telling us that a linear model was adequate, the common sense test for using the model caused major refinements until we got a nonlinear model that has good diagnostics and passed the common sense tests (Fox, 2011; 2012).

*Step 7*: Strengths and weaknesses.

No model is complete without self-reflection of the modeling process. We need to consider not only what we did right but also what we did that might be suspected as well as what we could do better. This reflection also helps in refining models in the future.

*Step 8*: Sensitivity analysis and model testing.

Sensitivity analysis is used to determine how *sensitive* a model is to changes in the value of the parameters of the model and to changes in the structure of the model.

Parameter sensitivity is usually performed as a series of tests in which the modeler sets different parameter values to see how a change in the parameter causes a change in the dynamic behavior of the stocks. By showing how the model behavior responds to changes in parameter values, sensitivity analysis is a useful tool in model building as well as in model evaluation.

Sensitivity analysis helps to build confidence in the model by studying the uncertainties that are often associated with parameters in models. Many parameters in system dynamics models represent quantities that are very difficult, or even impossible to measure to a great deal of accuracy in the real world. In addition, some parameter values change in the real world. Therefore, when building a system dynamics model, the modeler is usually at least somewhat uncertain about the parameter values he chooses and must use estimates. Sensitivity analysis allows him to determine what level of accuracy is necessary for a parameter to make the model sufficiently useful and valid. If the tests reveal that the model is insensitive, then it may be possible to use an estimate rather than a value with greater precision. Sensitivity analysis can also indicate which parameter values are reasonable to use in the model. If the model behaves as expected from real-world observations, it gives some indication that the parameter values reflect, at least in part, the *real world*. Sensitivity tests help the modeler to understand the dynamics of a system.

Experimenting with a wide range of values can offer insights into the behavior of a system in extreme situations. Discovering that the system behavior greatly changes for a change in a parameter value can identify a leverage point in the model—a parameter whose specific value can significantly influence the behavior mode of the system.

In our SIR example, we would test how the changes in the parameters affect the solution.

*Step 9*: Implement and maintain the model if it passes the common sense test.

A model is pointless if we do not use it. The more user-friendly the model, the more it will be used. Sometimes, the ease of obtaining data for the model can dictate the model's success or failure. The model must also remain current. Often this entails updating the data used for the model and updating the parameters used in the model.

### 1.2.5 Illustrative Examples: Starting the Modeling Process

We now demonstrate the modeling process that was presented in Section 1.2.4. Emphasis is placed on identifying the problem and on choosing appropriate (useable) variables in this section.

**Example 1.1: Prescribed Drug Dosage**

*Scenario*: Consider a patient who needs to take a newly marketed prescribed drug. To prescribe a safe and effective regimen for treating the disease, one must maintain a blood concentration above some effective level and below any unsafe level. How is this determined?

*Understanding the decision and problem*: Our goal is a mathematical model that relates dosage and time between dosages to the level of the drug in the bloodstream. What is the relationship between the amount of drug taken and the amount of drug in the blood after time, *t*? By answering this question, we are empowered to examine other facets of the problem of taking a prescribed drug.

*Assumptions*: We should choose or know the disease in question and the type (name) of the drug that is to be taken. We will assume in this example that the drug is rythmol, a drug taken for the regulation of the heartbeat. We need to know or find decaying rate of rythmol in the bloodstream. This might be found from data that have been previously collected. We need to find the safe and unsafe levels of rythmol based on the drug's *effects* within the body. This will serve as bounds for our model. Initially, we might assume that the patient size and weight have no effect on the drug's decay rate. We might assume that all patients are about the same size and weight. All are in good health and no one takes other drugs that affect the prescribed drug. We assume that all internal organs are functioning properly. We might assume that we can model this using a discrete time period even though the absorption rate is a continuous function. These assumptions help simplify the model.

**Example 1.2: Emergency Medical Response**

The Emergency Service Coordinator (ESC) for a county is interested in locating the county's three ambulances to maximize the residents that can be reached within 8 minutes in emergency situations. The county is divided into six zones, and the average time required to travel from one region to the next under semiperfect conditions is summarized in Table 1.1.

The population in zones 1, 2, 3, 4, 5, and 6 is given in Table 1.2.

*Understanding the decision and problem*: We want better coverage to improve the ability to take care of patients who require an ambulance to go to a hospital. Determine the location for placement of the ambulances to maximize coverage within the predetermined allotted time.

*Assumptions*: We initially assume that time travel between zones is negligible. We further assume that the times in the data are averages under ideal circumstances.

**TABLE 1.1**

Average Travel Times from Zone $i$ to Zone $j$ in Perfect Conditions

| | **1** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|---|
| **1** | 1 | 8 | 12 | 14 | 10 | 16 |
| **2** | 8 | 1 | 6 | 18 | 16 | 16 |
| **3** | 12 | 18 | 1.5 | 12 | 6 | 4 |
| **4** | 16 | 14 | 4 | 1 | 16 | 12 |
| **5** | 18 | 16 | 10 | 4 | 2 | 2 |
| **6** | 16 | 18 | 4 | 12 | 2 | 2 |

**TABLE 1.2**

Populations in Each Zone

| | |
|---|---|
| 1 | 50,000 |
| 2 | 80,000 |
| 3 | 30,000 |
| 4 | 55,000 |
| 5 | 35,000 |
| 6 | 20,000 |
| Total | 270,000 |

**Example 1.3: Bank Service Problem**

The bank manager is trying to improve customer satisfaction by offering better service. The management wants the average waiting time for the customer to be less than 2 minutes and the average length of the queue (length of the line waiting) to be two persons or fewer. The bank estimates about 150 customers per day. The existing arrival and service times are given in Tables 1.3 and 1.4.

*Understand the decision and problem*: The bank wants to improve customer satisfaction. First, we must determine if we are meeting the goal or not. Build a mathematical model to determine if the bank is meeting its goals and if not, come up with some recommendations to improve customer satisfaction.

*Assumptions*: Determine if the current customer service is satisfactory according to the manager guidelines. If not, determine the minimal changes for servers that are required to accomplish the manager's goal through modeling. We might begin by selecting a queuing model off the shelf to obtain some benchmark values.

**TABLE 1.3**

Arrival Times

| Time between Arrivals in Minutes | Probability |
|---|---|
| 0 | 0.10 |
| 1 | 0.15 |
| 2 | 0.10 |
| 3 | 0.35 |
| 4 | 0.25 |
| 5 | 0.05 |

**TABLE 1.4**

Service Times

| Service Time in Minutes | Probability |
|---|---|
| 1 | 0.25 |
| 2 | 0.20 |
| 3 | 0.40 |
| 4 | 0.15 |

Determine if the current customer service is satisfactory according to the manager guidelines. If not, determine the minimal changes for servers that are required to accomplish the manager's goal through modeling. We might begin by selecting a queuing model off the shelf to obtain some bench mark values.

**Example 1.4: Measuring Efficiency of Units**

We have three major units where each unit has two inputs and three outputs as shown in Table 1.5.

*Understand the decision and problem*: We want to improve efficiency of our operation. We want to be able to find *best practices* to share. First, we have to measure efficiency. We need to build a mathematic model to examine efficiency of a unit based on their inputs and outputs and be able to compare efficiency to other units.

*Assumptions and variable definitions*: We define the following decision variables:

$t_i$ is the value of a single unit of output of decision-making unit (DMU$i$), for $i = 1,2,3$

$w_i$ is the cost or weights for one unit of inputs of DMU$i$, for $i = 1,2$

$\text{Efficiency}_i$ = (total value of $i$'s outputs)/(total cost of $i$'s inputs), for $i = 1,2,3$

The following modeling initial assumptions are made:

1. No unit will have an efficiency more than 100%.
2. If any efficiency is less than 1, then it is inefficient.

**Example 1.5: World War II Battle of the Bismarck Sea**

In February 1943 at a critical stage of the struggle for New Guinea, the Japanese decided to bring reinforcements from the nearby island of New Britain. In moving their troops, the Japanese could either route north where rain and poor visibility were expected or south where clear weather was expected. In either case, the trip would be 3 days. Which route should they take? If the Japanese were only interested in time, they would be indifferent to the two routes. Perhaps, they wanted to minimize their convoy to expose by U.S. bombers. For the United States, General Kenney also faced a difficult choice. Allied intelligence had detected evidence of the Japanese convoy that assemble at the far side of New Britain. Kenney, of course, wanted to maximize the days that the bombers could attack the convoy, but he did not have enough reconnaissance planes to saturate both routes. What should he do?

**TABLE 1.5**

Input and Outputs

| Unit | Input #1 | Input #2 | Output #1 | Output #2 | Output #3 |
|---|---|---|---|---|---|
| 1 | 5 | 14 | 9 | 4 | 16 |
| 2 | 8 | 15 | 5 | 7 | 10 |
| 3 | 7 | 12 | 4 | 9 | 13 |

*Understand the decision and problem*: We want to build and use a mathematical model of conflict between players to determine the *best* strategy option for each player.

*Assumptions*: Let us assume that General Kenney can search only south or north. We will put these into rows. Let us further assume that the Japanese can actually sail north or south and let us put these in columns. Assume that we get additional information from the intelligence community of the U.S. Armed Forces, and that this information is accurate. This information states that if there is a clear exposure, then we bomb all 3 days. If we search south and do not find the enemy (then have to search north in the poorer weather that will waste 2 days of searching), then we have only 1 day to bomb. If we search north and Japanese sail north, the enemy will be exposed in 2 days. If we search north and the Japanese sail south, the enemy will be exposed in 2 days.

**Example 1.6: Risk Analysis for Homeland Security**

Consider providing support to the Department of Homeland Security. The department only has so many assets and a finite amount of time to conduct investigations, thus priorities might be established. The risk assessment office has collected the data for the morning meeting as shown in Table 1.6. Your operations research team must analyze the information and must provide a priority list to the risk assessment team for that meeting.

*Understand the decision and problem*: There are more risks than we can possibly investigate. Perhaps if we rank these based on useful criteria, we can determine a priority for investigating these risks. We need to construct a useful mathematical model that ranks the incidents or risks in a priority order.

*Assumptions*: We have past decision that will give us insights into the decision maker's process. We have data only on reliability, approximate number of deaths, approximate costs to fix or rebuild, location, destructive influence, and on number of intelligence gathering tips. These will be the criteria for our analysis. The data are accurate and precise. We can convert word data into ordinal numbers.

*Model*: We could use multiattribute decision-making techniques for our model. We decide on a hybrid approach of analytical hierarchy process (AHP) and technique for order of preference by similarity to ideal solution (TOPSIS), that is, AHP-TOPSIS. We will use AHP with Saaty's (1980) pairwise comparison to obtain the decision-maker weights. We will also use the pairwise comparison to obtain numerical values for the criteria: location and destructive influence. Then we will use TOPSIS.

**TABLE 1.6**

Risk Assessment Priority

| Threat Alternatives/ Criterion | Reliability of Threat Assessment | Approximate Associated Deaths (000) | Cost to Fix Damages in (Millions) | Location | Destructive Psychological Influence | Number of Intelligence-Related Tips |
|---|---|---|---|---|---|---|
| 1. Dirty bomb threat | 0.40 | 10 | 150 | Urban dense | Extremely intense | 3 |
| 2. Anthrax-bioterror threat | 0.45 | .8 | 10 | Urban dense | Intense | 12 |
| 3. DC-road and bridge network threat | 0.35 | 0.005 | 300 | Urban & rural | Strong | 8 |
| 4. NY subway threat | 0.73 | 12 | 200 | Urban dense | Very strong | 5 |
| 5. DC metro threat | 0.69 | 11 | 200 | Both Urban dense and rural | Very strong | 5 |
| 6. Major bank robbery | 0.81 | 0.0002 | 10 | Urban dense | Weak | 16 |
| 7. FAA threat | 0.70 | 0.001 | 5 | Rural dense | Moderate | 15 |

**Example 1.7: Discrete SIR Models of Epidemics**

Consider a disease that is spreading throughout the United States such as the new deadly flu. The Centers for Disease Control and Prevention is interested in knowing and experimenting with a model for this new disease before it actually becomes a *real* epidemic. Let us consider the population that is beingdivided into three categories: susceptible, infected, and removed. We make the following assumptions for our model:

- No one enters or leaves the community, and there is no contact outside the community.
- Each person is either susceptible, $S$ (able to catch this new flu); infected, $I$ (currently has the flu and can spread the flu); or removed, $R$ (already had the flu and will not get it again that includes death).
- Initially every person is either $S$ or $I$.
- Once someone gets the flu this year, they will not get again.
- The average length of the disease is 2 weeks over which the person is deemed to be infected and can spread the disease.
- Our time period for the model will be per week.

The model we will consider is an off-the-shelf model, the SIR model (Allman and Rhodes, 2004).

Let us assume the following definition for our variables:

$S(n)$ is the number in the population susceptible after period $n$.

$I(n)$ is the number infected after period $n$.

$R(n)$ is the number removed after period $n$.

Let us start our modeling process with $R(n)$. Our assumption for the length of time someone has the flu is 2 weeks. Thus, half of the infected people will be removed each week:

$$R(n+1) = R(n) + 0.5I(n)$$

The value, 0.5, is called the removal rate per week. It represents the proportion of the infected persons who are removed from infection each week. If real data are available, then we could do *data analysis* to obtain the removal rate.

$I(n)$ will have terms that both increase and decrease its amount over time. It is decreased by the number that is removed each week, $0.5 * I(n)$. It is increased by the number of susceptible people who come into contact with an infected person and catch the disease, $aS(n)I(n)$. We define $a$ as the rate at which the disease is spread or as the transmission coefficient. We realize that this is a probabilistic coefficient. We will assume, initially, that this rate is a constant value that can be found from initial conditions.

Let us illustrate as follows: Assume that we have a population of 1000 students in the dorms. Our nurse found that three students were reporting to the infirmary initially. The next week, five students came to the infirmary with flu-like symptoms. $I(0) = 3$, $S(0) = 997$. In week 1, the number of newly infected students is 30.

$$5 = a\ I(n)S(n) = a(3)*(995)$$

$$a = 0.00167$$

Let us consider $S(n)$. This number is decreased only by the number that becomes infected. We may use the same rate, $a$, as before to obtain the model:

$$S(n+1) = S(n) - aS(n)I(n)$$

Our coupled SIR model is

$$R(n+1) = R(n) + 0.5I(n)$$

$$I(n+1) = I(n) - 0.5I(n) + 0.00167I(n)S(n)$$

$$S(n+1) = S(n) - 0.00167S(n)I(n)$$

$$I(0) = 3, S(0) = 997, R(0) = 0$$

The SIR model can be solved iteratively and can be viewed graphically. We will revisit this again in Chapter 7. In Chapter 7, we determine that the worse of the flu epidemic occurs around week 8, at the maximum of the infected graph. The maximum number is slightly larger than 400; from Figures 7.30–7.33 in Chapter 7, it is approximated as 427. After 25 weeks, slightly more than 9 people never got the flu.

These examples will be solved in subsequent chapters.

## 1.3 Technology

Most real-world problems that we have been involved in solving model require technology to assist the analyst, the modeler, and the decision-maker. Microsoft Excel is available on most computers and represents a fairly good technological support for analysis of the average problems, especially with *Analysis ToolPak* and the *Solver* installed. Other specialized software to assist analysts include MATLAB®, Maple, Mathematica, LINDO, LINGO, GAMS, as well as some additional add-ins for Excel such as the simulation package and Crystal Ball. Analysts should avail themselves to have access to as many of these packages as necessary. In this book, we illustrate Excel and Maple although other software may be easily substituted.

## 1.4 Conclusion

We have provided a clear and simple process to begin mathematical modeling in applied situation that requires the stewardship of applied mathematics, operations research analysis, or risk assessment. We did not cover all the possible models but highlighted a few through illustrative examples. We emphasize that sensitivity analysis is extremely important in all models and should be accomplished before making any decision. We show this in more detail in the following chapters that cover the techniques.

### Exercises

Using Steps 1–3 of the modeling process, identify a problem from scenario 1–11 that you could study. There are no *right* or *wrong* answers, just measures of difficulty.

1. The population of deer in your community.
2. A new outdoor shopping mall is being constructed. How should you design the illumination of the parking lot?
3. A farmer wants a successful season with his crops. He thinks that this is accomplished by growing a lot of anything as long as he uses all his land.
4. Ford Motor Company bought Volvo. Are Volvos still *top quality* cars?
5. A minor Forbes 500 company wants to go mobile with internet access and computer upgrades, but cost might be a problem.
6. Starbucks has many varieties of coffee available. How can Starbucks make more money?
7. A student does not like math or math-related courses. How can a student maximize their chances for a good grade in a math class to improve their overall GPA?
8. Freshmen think that their first semester courses should be pass–fail for credit or no credit only.
9. Alumni are clamoring to fire the college's head football coach.
10. Stocking a fish pond with bass and trout.
11. Safety airbags in millions of cars are to be replaced. Can this be done in a timely manner?

### Projects

1. Is Michael Jordan or Stephan Curry the greatest basketball player of the century? What variables and factors need to be considered?

2. What kind of car should you buy when you graduate from college? What factors should be in your decision? Are car companies modeling your needs?
3. Consider domestic decaffeinated coffee brewing. Suggest some objectives that could be used if you wanted to market your new brew. What variables and data would be useful?
4. Replacing a coaching legend at a school is a difficult task. How would you model this? What factors and data would you consider? Would you equally weigh all factors?
5. How would you go about building a model for the *best pro football player of all time*?
6. Rumors abound in major league baseball about steroid use. How would you go about creating a model that could imply the use of steroids?

## References and Suggested Readings

Albright, B. 2010. *Mathematical Modeling with Excel*. Sudbury, MA: Jones and Bartlett Publishers.

Allman, E. and J. Rhodes. 2004. *Mathematical Modeling with Biology*: *An Introduction*. Cambridge, UK: Cambridge University Press.

Bender, E. 2000. Chapter 1, *Mathematical Modeling*. Mineola, NY: Dover Press, pp. 1–8.

Fox, W. P. 2011. Using the EXCEL solver for nonlinear regression. *Computers in Education Journal*, 2(4), 77–86.

Fox, W. P. (2012). Issues and importance of "good" starting points for nonlinear regression for mathematical modeling with maple: Basic model fitting to make predictions with oscillating data. *Journal of Computers in Mathematics and Science Teaching*, 31(1), 1–16.

Fox, W. P. 2013a. Modeling engineering management decisions with game theory. In F. P. García Márquez and B. Lev (Eds.), *Engineering Management*. Rijeka, Croatia: InTech.

Fox, W. 2013b. Mathematical modeling and analysis: An example using a Catapult. *Computer in Education Journal*, 4(3): 69–77.

Fox, W. 2014a. Chapter 17, Game theory in business and industry. In *Encyclopedia of Business Analytics and Optimization*. Hershey, PA: IGI Global and Sage Publications, V(1), pp. 162–173.

Fox, W. 2014b. Chapter 221, TOPSIS in business analytics. In *Encyclopedia of Business Analytics and Optimization*. Hershey, PA: IGI Global and Sage Publications, V(5), pp. 281–291.

Fox, W. P. 2016. Applications and modeling using multi-attribute decision making to rank terrorist threats. *Journal of Socialomics*, 5(2), 1–12.

Fox, W. P. and M. J. Jaye. 2011. Search and rescue carry through mathematical modeling problem. *Computers in Education Journal*, 2(3), 82–93.

Giordano, F. R., W. Fox, and S. Horton. 2014. *A First Course in Mathematical Modeling* (5th ed.). Boston, MA: Brooks-Cole Publishers.
Meerschaert, M. M. 1999. *Mathematical Modeling* (2nd ed.). San Diego, CA: Academic Press.
Myer, W. 2004. *Concepts of Mathematical Modeling*. New York: Dover Press, pp. 13–15.
Saaty, T. 1980. *The Analytical Hierarchy Process*. New York: McGraw Hill.
Winston, W. 1995. *Introduction to Mathematical Programming.* Belmont, CA: Duxbury Press, pp. 323–325.

# 2

# *Introduction to Stochastic Decision-Making Models for Business Analytics*

**OBJECTIVES**

1. Know the basic concepts of probability and expected value.
2. Understand and use expected value in decision-making under risk and uncertainty.
3. Apply Bayesian probability to decision-making.
4. Develop a decision tree to compare alternatives.
5. Be able to use appropriate technology.

Consider a company that needs to decide whether they should build and market large or small outdoor play sets. The three alternatives under consideration with their respective demand revenues and losses under the estimated demand probabilities are as follows:

| | Outcomes | | |
|---|---|---|---|
| | **High Demand** | **Moderate Demand** | **Low Demand** |
| Alternatives | ($p_{hd}$ = 0.35) | ($p_{md}$ = 0.40) | ($p_{ld}$ = 0.25) |
| Large play sets | \$200,000 | \$120,000 | −\$125,000 |
| Small play sets | \$100,000 | \$55,000 | −\$25,000 |
| No play sets | \$0 | \$0 | \$0 |

Can we determine the best course of action for this company? We will analyze and solve this problem later in this chapter.