Name _____ Class _____ Date _____

Practice

Form G

Polynomial Functions

Write each polynomial in standard form. Then classify it by degree and by number of terms.

1. $4x + x + 2$
   $5x + 2$; linear binomial
2. $-3 + 3x - 3x$
   -3; constant monomial
3. $6x^4 - 1$
   $6x^4 - 1$; quartic binomial
4. $1 - 2s + 5s^4$
   $5s^4 - 2s + 1$; quartic trinomial
5. $5m^2 - 3m^2$
   $2m^2$; quadratic monomial
6. $x^2 + 3x - 4x^3$
   $-4x^3 + x^2 + 3x$; cubic trinomial
7. $-1 + 2x^2$
   $2x^2 - 1$; quadratic binomial
8. $5m^2 - 3m^3$
   $-3m^3 + 5m^2$; cubic binomial
9. $5x - 7x^2$
   $-7x^2 + 5x$; quadratic binomial
10. $2 + 3x^3 - 2$
    $3x^3$; cubic monomial
11. $6 - 2x^3 - 4 + x^3$
    $-x^3 + 2$; cubic binomial
12. $6x - 7x$
    $-x$; linear monomial
13. $a^3(a^2 + a + 1)$
    $a^5 + a^4 + a^3$; quintic trinomial
14. $x(x + 5) - 5(x + 5)$
    $x^2 - 25$; quadratic binomial
15. $p(p - 5) + 6$
    $p^2 - 5p + 6$; quadratic trinomial
16. $(3c^2)^2$
    $9c^4$; quartic monomial
17. $-(3 - b)$
    $b - 3$; linear binomial
18. $6(2x - 1)$
    $12x - 6$;linear binomial
19. $\frac{2}{3} + s^2$
    $s^2 + \frac{2}{3}$; quadratic binomial
20. $\frac{2x^4 + 4x - 5}{4}$
    $\frac{1}{2}x^4 + x - \frac{5}{4}$; quartic trinomial
21. $\frac{3 - z^5}{3}$
    $-\frac{1}{3}z^5 + 1$; quintic binomial

Determine the end behavior of the graph of each polynomial function.

22. $y = 3x^4 + 6x^3 - x^2 + 12$
    up and up
23. $y = 50 - 3x^3 + 5x^2$
    up and down
24. $y = -x + x^2 + 2$
    up and up
25. $y = 4x^2 + 9 - 5x^4 - x^3$
    down and down
26. $y = 12x^4 - x + 3x^7 - 1$
    down and up
27. $y = 2x^5 + x^2 - 4$
    down and up
28. $y = 5 + 2x + 7x^2 - 5x^3$
    up and down
29. $y = 20 - 5x^6 + 3x - 11x^3$
    down and down
30. $y = 6x + 25 + 4x^4 - x^2$
    up and up

Describe the shape of the graph of each cubic function by determining the end behavior and number of turning points.

31. $y = x^3 + 4x$
    down and up; no turns
32. $y = -2x^3 + 3x - 1$
    up and down; two turns
33. $y = 5x^3 + 6x^2$
    down and up; two turns

Determine the degree of the polynomial function with the given data.

34. 3rd degree

x	y
−2	−16
−1	1
0	4
1	5
2	16

35. 4th degree

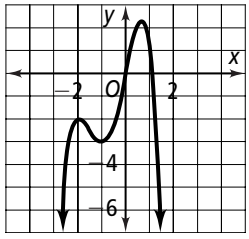
x	y
−2	52
−1	6
0	2
1	4
2	48

Name _____ Class _____ Date _____

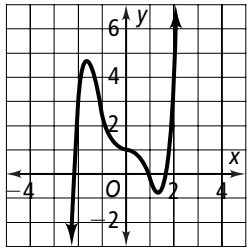
Practice (continued) *Form G*

Polynomial Functions

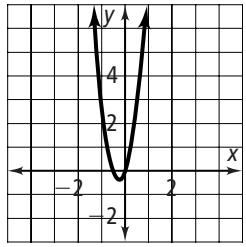
Determine the sign of the leading coefficient and the degree of the polynomial function for each graph.

36. 

negative; 4th degree

37. 

positive; 5th degree

38. 

positive; 2nd degree

39. Error Analysis A student claims the function $y = 3x^4 - x^3 + 7$ is a fourth-degree polynomial with end behavior of down and down. Describe the error the student made. What is wrong with this statement? **The degree is even and the leading coefficient is positive, so the end behavior should be up and up.**

40. The table at the right shows data representing a polynomial function.

a. What is the degree of the polynomial function? **5th degree**

b. What are the second differences of the y-values? **−726, −126, −6, 114, 714**

c. What are the differences when they are constant? **480**

x	y
−3	−999
−2	−140
−1	−7
0	0
1	1
2	116
3	945

Classify each polynomial by degree and by number of terms. Simplify first if necessary.

41. $4x^5 - 5x^2 + 3 - 2x^2$
5th degree; 3 terms

42. $b(b-3)^2$
3rd degree; 3 terms

43. $(7x^2 + 9x - 5) + (9x^2 - 9x)$
2nd degree; 2 terms

44. $(x+2)^3$
3rd degree; 4 terms

45. $(4s^4 - s^2 - 3) - (3s - s^2 - 5)$
4th degree; 3 terms

46. 13
0 degree; 1 term

47. Open-Ended Write a third-degree polynomial function. Make a table of values and a graph. **Check students' work.**

48. Writing Explain why finding the degree of a polynomial is easier when the polynomial is written in standard form. **When a polynomial is written in standard form, the term with the greatest exponent becomes the first term. This exponent is equal to the degree of the polynomial.**

Name _____ Class _____ Date _____

Practice

Form G

Adding, Subtracting, and Multiplying Polynomials

Simplify each product.

1. $2x(x + 8)$ **$2x^2 + 16x$**
2. $(n + 7)5n$ **$5n^2 + 35n$**
3. $6h^2(7 + h)$ **$6h^3 + 42h^2$**
4. $-b^2(b - 10)$ **$-b^3 + 10b^2$**
5. $-3c(8 + 2c - c^3)$ **$3c^4 - 6c^2 - 24c$**
6. $y(2y^2 - 3y + 6)$ **$2y^3 - 3y^2 + 6y$**
7. $4t(t^2 - 6t + 2)$ **$4t^3 - 24t^2 + 8t$**
8. $-m(4m^3 - 8m^2 + m)$ **$-4m^4 + 8m^3 - m^2$**
9. $7j(-2j^2 - 8j - 3)$ **$-14j^3 - 56j^2 - 21j$**
10. $-t^2(2t^4 + 4t - 8)$ **$-2t^6 - 4t^3 + 8t^2$**
11. $2k(-3k^3 + k^2 - 10)$ **$-6k^4 + 2k^3 - 20k$**
12. $8a^2(-a^7 + 7a - 7)$ **$-8a^9 + 56a^3 - 56a^2$**
13. $4v^3(2v^2 - 3v + 5)$ **$8v^5 - 12v^4 + 20v^3$**
14. $5d(-d^3 + 2d^2 - 3d)$ **$-5d^4 + 10d^3 - 15d^2$**
15. $11w(w^2 + 2w + 6)$ **$11w^3 + 22w^2 + 66w$**
16. $2a^3b + 4a^3b$ **$6a^3b$**
17. $5x^3 - 4x^3$ **x^3**
18. $3m^6n^3 - 5m^6n^3$ **$-2m^6n^3$**
19. $-6ab + 3ab$ **$-3ab$**
20. $4c^2d^6 - 7c^2d^6$ **$-3c^2d^6$**
21. $315x^2 - 30x^2$ **$285x^2$**
22. $8z - 12$
    $+ 6z + 9$
    $14z - 3$
23. $9x^3 + 3$
    $+ 4x^3 + 7$
    $13x^3 + 10$
24. $6j^2 - 2j + 5$
    $+ 3j^2 + 4j - 6$
    $9j^2 + 2j - 1$
25. $(3k^2 + 5) + (16x^2 + 7)$ **$3k^2 + 16x^2 + 12$**
26. $(g^4 - 4g^2 + 11) + (-g^3 + 8g)$ **$g^4 - g^3 - 4g^2 + 8g + 11$**
27. A local deli kept track of the sandwiches it sold for three months. The polynomials below model the number of sandwiches sold, where s represents days.

    Ham and Cheese: $4s^3 - 28s^2 + 33s + 250$
    Pastrami: $-7.4s^2 + 32s + 180$

    Write a polynomial that models the total number of these sandwiches that were sold. **$4s^3 - 35.4s^2 + 65s + 430$**

Simplify.

28. $11n - 4$
    $-(5n + 2$
    $6n - 6$
29. $7x^4 + 9$
    $-(8x^4 + 2$
    $-x^4 + 7$
30. $3d^2 + 8d - 2$
    $-(2d^2 - 7d + 6)$
    $d^2 + 15d - 8$
31. $(28e^3 + 3e^2) + (19e^3 + e^2)$ **$47e^3 + 4e^2$**
32. $(-12h^4 + h) - (-6h^4 + 3h^2 - 4h)$ **$-6h^4 - 3h^2 + 5h$**

Name _____ Class _____ Date _____

Practice (continued) *Form G*

Adding, Subtracting, and Multiplying Polynomials

33. A small town wants to compare the number of students enrolled in public and private schools. The polynomials below show the enrollment for each:

Public School: $-19c^2 + 980c + 48{,}989$
Private School: $40c + 4046$

Write a polynomial for how many more students are enrolled in public school than private school. $-19c^2 + 940c + 44{,}943$

Simplify. Write each answer in standard form.

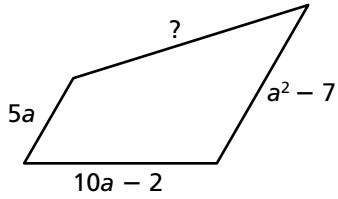
34. $(3a^2 + a + 5) - (2a - 5)$
$3a^2 - a + 10$

35. $(6d - 10d^3 + 3d^2) - (5d^3 + 3d - 4)$
$-15d^3 + 3d^2 + 3d + 4$

36. $(-4s^3 + 2s - 3) + (-2s^2 + s + 7)$
$-4s^3 - 2s^2 + 3s + 4$

37. $(8p^3 - 6p + 2p^2) + (9p^2 - 5p - 11)$
$8p^3 + 11p^2 - 11p - 11$

38. The fence around a quadrilateral-shaped pasture is $3a^2 + 15a + 9$ long. Three sides of the fence have the following lengths: $5a$, $10a - 2$, $a^2 - 7$. What is the length of the fourth side of the fence?
$2a^2 + 18$



39. Error Analysis Describe and correct the error in simplifying the sum shown at the right.
two unlike terms, $6x^3$ and $-3x^2$, were added; $6x^3 - 3x^2 + 6x - 2$

$$\begin{array}{r} 6x^3 + 4x - 10 \\ +\ (-3x^2 + 2x + 8) \\ \hline 3x^3 + 6x - 2 \end{array}$$

Simplify. Write in standard form.

40. $-3x(4x^2 - 6x + 12)$
$-12x^3 + 18x^2 - 36x$

41. $-7y^2(-4y^3 + 6y)$
$28y^5 - 42y^3$

42. $9a(-3a^2 + a - 5)$
$-27a^3 + 9a^2 - 45a$

43. $p(p + 4) - 2p(p - 8)$
$-p^2 + 20p$

44. $t(t + 4) - t(4t^2 - 2)$
$-4t^3 + t^2 + 6t$

45. $6c(2c^2 - 4) - c(8c)$
$12c^3 - 8c^2 - 24c$

46. $-5m(2m^3 - 7m^2 + m)$
$-10m^4 + 35m^3 - 5m^2$

47. $2q(q + 1) - q(q - 1)$
$q^2 + 3q$

48. $-n^2(-6n^2 + 2n)$
$6n^4 - 2n^3$

49. A cylinder has a radius of $3m^2n$ and a height of $7mn$. The formula for the volume of a cylinder is $V = \pi r^2 h$, where r is the radius and h is the height. What is the volume of the cylinder? Simplify your answer. $63\pi m^5 n^3$

Name _____ Class _____ Date _____

Practice

Form G

Polynomials, Linear Factors, and Zeros

Write each polynomial in factored form. Check by multiplication.

1. $2x^3 + 10x^2 + 12x$ $2x(x + 2)(x + 3)$

2. $x^4 - x^3 - 6x^2$ $x^2(x + 2)(x - 3)$

3. $-3x^3 + 18x^2 - 27x$ $-3x(x - 3)^2$

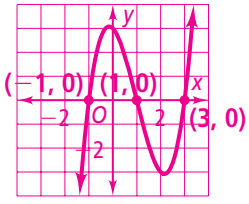
4. $x^3 - 2x^2 + x$ $x(x - 1)^2$

5. $x^3 + 7x^2 + 15x + 9$ $(x + 3)^2(x + 1)$

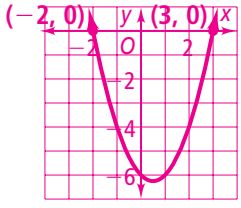
6. $2x^4 + 23x^3 + 60x^2 - 125x - 500$ $(x + 4)(2x - 5)(x + 5)^2$

Find the zeros of each function. Then graph the function.

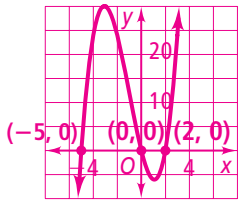
7. $y = (x + 1)(x - 1)(x - 3)$
   $-1, 1, 3$



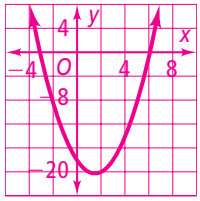
8. $y = (x + 2)(x - 3)$
   $-2, 3$



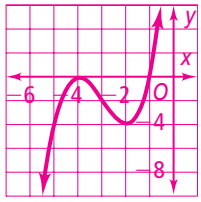
9. $y = x(x - 2)(x + 5)$
   $-5, 0, 2$



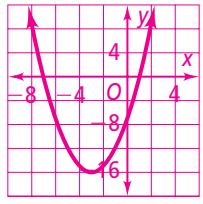
10. $y = (x - 6)(x + 3)$
    $6, -3$



11. $y = (x + 4)^2(x + 1)$
    $-4, -4, -1$



12. $y = (x - 1)(x + 7)$
    $1, -7$



Write a polynomial function in standard form with the given zeros.

13. $x = -1, 3, 4$
    $y = x^3 - 6x^2 + 5x + 12$

14. $x = 1, 1, 2$
    $y = x^3 - 4x^2 + 5x - 2$

15. $x = -3, 0, 0, 5$
    $y = x^4 - 2x^3 - 15x^2$

16. $x = 4, 2, -3, 0$
    $y = x^4 - 3x^3 - 10x^2 + 24x$

17. $x = -1, 5, -2$
    $y = x^3 - 2x^2 - 13x - 10$

18. $x = -6, 0$
    $y = x^2 + 6x$

Find the zeros of each function. State the multiplicity of multiple zeros.

19. $y = (x - 5)^3$
    5, multiplicity 3

20. $y = x(x - 8)^2$
    0; 8, multiplicity 2

21. $y = (x - 2)(x + 7)^3$
    2; −7, multiplicity 3

22. $y = x^4 - 8x^3 + 16x^2$
    0, multiplicity 2; 4, multiplicity 2

23. $y = 9x^3 - 81x$
    $-3, 0, 3$

24. $y = (2x + 5)(x - 3)^2$
    $-\frac{5}{2}$; 3, multiplicity 2

Name _____ Class _____ Date _____

Practice (continued) *Form G*

Polynomials, Linear Factors, and Zeros

Find the relative maximum and relative minimum of the graph of each function.

25. $f(x) = x^3 - 7x^2 + 10x$
rel. max.: 4.06; rel. min.: −8.21

26. $f(x) = x^3 - x^2 - 9x + 9$
rel. max.: 16.9; rel. min.: −5.05

27. $f(x) = x^4 + x^3 - 3x^2 - 5x - 2$
rel. min.: about −8.54

28. $f(x) = x^2 - 6x + 9$
rel. min.: 0

29. A rectangular box has a square base. The combined length of a side of the square base, and the height is 20 in. Let x be the length of a side of the base of the box.

a. Write a polynomial function in factored form modeling the volume V of the box. $V = x^2(20 - x)$

b. What is the maximum possible volume of the box? about 1185 in.^{3}

30. Reasoning A polynomial function has a zero at $x = -2a$. Find one of its factors. $(x + 2a)$

31. The side of a cube measures $3x + 2$ units long. Express the volume of the cube as a polynomial. $27x^3 + 54x^2 + 36x + 8$

32. Writing The volume of a box is $x^3 - 3x^2 + 3x - 1$ cubic units. Explain how to find the length of a side if the box is a cube. The factors of this polynomial are $(x - 1)(x - 1)(x - 1)$. Because all three factors are equal, and the sides of the box are equal, the side length would be $x - 1$.

33. You have a block of wood that you want to use to make a sculpture. The block is currently $3x$ units wide, $4x$ units high, and $5x$ units deep. You need to remove 1 unit from each dimension before you can begin your sculpture.

a. What is the original volume of the block? $60x^3$ cubic units

b. What is the new volume of the block? $60x^3 - 47x^2 + 12x - 1$ cubic units

c. What is the volume of the wood that you remove? $47x^2 - 12x + 1$ cubic units

34. What are the zeros and the multiplicity of each zero for the polynomial function $x^4 - 2x^2 + 1$? 1 of multiplicity 2, −1 of multiplicity 2

35. Error Analysis On your homework, you wrote that the polynomial function from the given zeros $x = 3, 0, -9, 1$ is $y = x^4 + 5x^3 - 33x^2 + 27x$. Your friend wrote that the polynomial function is $y = x^3 + 5x^2 - 33x + 27$. Who is correct? What mistake was made?
You are correct. Your friend did not include the factor with the zero $x = 0$ from the polynomial. It should include the factor $(x - 0)$.

Name _____ Class _____ Date _____

Practice

Form G

Solving Polynomial Equations

Find the real or imaginary solutions of each equation by factoring.

1. $8x^3 - 27 = 0$
   $(2x - 3)(4x^2 + 6x + 9)$; $\frac{3}{2}, \frac{-3 \pm 3i\sqrt{3}}{4}$
2. $x^3 + 64 = 0$
   $(x + 4)(x^2 - 4x + 16)$; $-4, 2 \pm 2i\sqrt{3}$
3. $2x^3 + 54 = 0$
   $2(x + 3)(x^2 - 3x + 9)$; $-3, \frac{3 \pm 3i\sqrt{3}}{2}$
4. $2x^3 - 250 = 0$
   $2(x - 5)(x^2 + 5x + 25)$; $5, \frac{-5 \pm 5i\sqrt{3}}{2}$
5. $4x^3 - 32 = 0$
   $4(x - 2)(x^2 + 2x + 4)$; $2, -1 \pm i\sqrt{3}$
6. $27x^3 + 1 = 0$
   $(3x + 1)(9x^2 - 3x + 1)$; $-\frac{1}{3}, \frac{1 \pm i\sqrt{3}}{6}$
7. $64x^3 - 1 = 0$
   $(4x - 1)(16x^2 + 4x + 1)$; $\frac{1}{4}, \frac{-1 \pm i\sqrt{3}}{8}$
8. $x^3 - 27 = 0$
   $(x - 3)(x^2 + 3x + 9)$; $3, \frac{-3 \pm 3i\sqrt{3}}{2}$
9. $x^4 - 5x^2 + 4 = 0$
   $(x + 1)(x - 1)(x + 2)(x - 2)$; $-2, -1, 1, 2$
10. $x^4 - 12x^2 + 11 = 0$
    $(x + 1)(x - 1)(x^2 - 11)$; $-1, 1, -\sqrt{11}, \sqrt{11}$
11. $x^4 - 10x^2 + 16 = 0$
    $(x^2 - 2)(x^2 - 8)$ $-\sqrt{2}, \sqrt{2}, -\sqrt{8}, \sqrt{8}$ $= 2\sqrt{2}$
12. $x^4 - 8x^2 + 16 = 0$
    $(x + 2)^2(x - 2)^2$; $-2, 2$
13. $x^4 - 9x^2 + 14 = 0$
    $(x^2 - 7)(x^2 - 2)$; $-\sqrt{7}, \sqrt{7}, -\sqrt{2}, \sqrt{2}$
14. $x^4 + 13x^2 + 36 = 0$
    $(x^2 + 4)(x^2 + 9)$; $-2i, 2i, -3i, 3i$
15. $x^4 - 10x^2 + 9 = 0$
    $(x + 1)(x - 1)(x + 3)(x - 3)$; $-1, 1, -3, 3$
16. $x^4 + 3x^2 - 4 = 0$
    $(x + 1)(x - 1)(x^2 + 4)$; $-1, 1, -2i, 2i$

Find the real solutions of each equation by graphing.

17. $2x^4 = 9x^2 - 4$ $-2, 2, -0.71, 0.71$
18. $x^2 - 16x = -1$ $0.06, 15.94$
19. $6x^3 + 10x^2 + 5x = 0$ 0
20. $36x^3 + 6x^2 = 9x$ $-0.59, 0, 0.42$
21. $15x^4 = 11x^3 + 14x^2$ $-0.67, 0, 1.4$
22. $x^4 = 81x^2$ $-9, 0, 9$

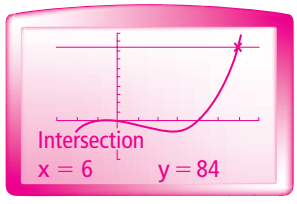
For Exercises 23 and 24, write an equation to model each situation. Then solve each equation by graphing.

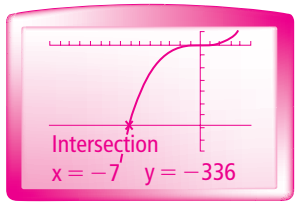
23. The volume V of a container is 84 ft^3. The width, the length, and the height are x, $x + 1$, and $x - 4$ respectively. What are the container's dimensions?

24. The product of three consecutives integers $n - 1$, n, and $n + 1$ is -336. What are the integers?

24. $(n - 1)(n)(n + 1) = -336$; $-8, -7, -6$

23. $x^3 - 3x^2 - 4x = 84$
    $x = 6$ ft
    $x + 1 = 7$ ft
    $x + 4 = 2$ ft





Name _____ Class _____ Date _____

Practice (continued) *Form G*

Solving Polynomial Equations

Solve each equation.

25. $x^4 - x = 0$

0, 1, $\frac{-1 \pm i\sqrt{3}}{2}$

26. $3x^4 + 18 = 21x^2$

$-1, 1, -\sqrt{6}, \sqrt{6}$

27. $2x^4 - 26x^2 - 28 = 0$

$-\sqrt{14}, \sqrt{14}, -i, i$

28. $5x^4 + 50x^2 + 80 = 0$

$-i\sqrt{2}, i\sqrt{2}, -2i\sqrt{2}, 2i\sqrt{2}$

29. $x^4 - 81 = 0$

$-3, 3, -3i, 3i$

30. $x^4 = 25$

$-\sqrt{5}, \sqrt{5}, -i\sqrt{5}, i\sqrt{5}$

31. $x^5 = x^3 + 12x$

$0, -2, 2, -i\sqrt{3}, i\sqrt{3}$

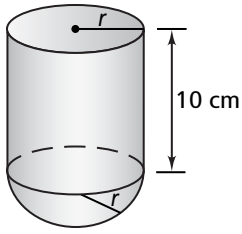
32. $x^4 + 12x^2 = 8x^3$

0, 2, 6

33. Over 3 years, you save your earnings from a summer job. The polynomial $1600x^3 + 1200x^2 + 800x$ represents your savings, with interest, at the end of the 3 years. The annual interest rate equals $x - 1$. Find the interest rate needed so that you will have \$4000 at the end of 3 years. **4.82%**

34. Error Analysis Your friend claims that the zeros of $3x^3 + 7x^2 - 22x - 8 = 0$ are -4, 2, and -1. What did your friend do wrong? What are the correct factors? **Your friend forgot to divide by 3 when solving an equation to find the third factor. The correct factors are -4, 2, and $-\frac{1}{3}$.**

35. The container at the right consists of a cylinder on top of a hemisphere. The container holds 500 cm^3. What is the radius of the container, to the nearest hundredth of a centimeter? **3.58 cm**



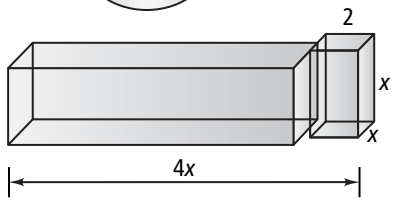
36. Suppose a 2-in. slice is cut from one face of the cheese block as shown. The remaining block has a volume of 224 $in.^3$.

a. What are the dimensions of the new block? **4 in. × 4 in. × 14 in.**

b. What are the dimensions of the old block? **4 in. × 4 in. × 16 in.**

c. What is the original volume? **256 $in.^3$**

d. What is the volume of the cut slice? **32 $in.^3$**



37. Reasoning A test question asks you to find three integers whose product is 412. Do you have enough information to solve this problem? Explain. **Yes, but there are multiple solutions.**

38. Your mother is 25 years older than you. Your father is 3 years older than your mother. The product of all three ages is 32,130. How old is your father? **45 years old**

Name _____ Class _____ Date _____

Practice

Form G

Dividing Polynomials

Divide using long division. Check your answers.

1. $(x^2 - 13x - 48) \div (x + 3)$
   $x - 16$
2. $(2x^2 + x - 7) \div (x - 5)$
   $2x + 11$, R 48
3. $(x^3 + 5x^2 - 3x - 1) \div (x - 1)$
   $x^2 + 6x + 3$, R 2
4. $(3x^3 - x^2 - 7x + 6) \div (x + 2)$
   $3x^2 - 7x + 7$, R -8
5. $(x^2 - 3x + 1) \div (x - 4)$
   $x + 1$, R 5
6. $(x^3 - 4x^2 + 3x + 2) \div (x + 2)$
   $x^2 - 6x + 15$, R -28

Determine whether each binomial is a factor of $x^3 + 3x^2 - 10x - 24$.

7. $x + 4$ yes
8. $x - 3$ yes
9. $x + 6$ no
10. $x + 2$ yes

Divide using synthetic division.

11. $(x^3 - 8x^2 + 17x - 10) \div (x - 5)$
    $x^2 - 3x + 2$
12. $(x^3 + 5x^2 - x - 9) \div (x + 2)$
    $x^2 + 3x - 7$, R 5
13. $(-2x^3 + 15x^2 - 22x - 15) \div (x - 3)$
    $-2x^2 + 9x + 5$
14. $(x^3 + 7x^2 + 15x + 9) \div (x + 1)$
    $x^2 + 6x + 9$
15. $(x^3 + 2x^2 + 5x + 12) \div (x + 3)$
    $x^2 - x + 8$, R -12
16. $(x^3 - 5x^2 - 7x + 25) \div (x - 5)$
    $x^2 - 7$, R -10
17. $(x^4 - x^3 + x^2 - x + 1) \div (x - 1)$
    $x^3 + x$, R 1
18. $(2x^4 + 7x^3 - 11x^2 + 21x + 5) \div (x + 5)$
    $2x^3 - 3x^2 + 4x + 1$
19. $(x^4 - 5x^3 + 5x^2 + 7x - 12) \div (x - 4)$
    $x^3 - x^2 + x + 11$, R 32
20. $(2x^4 + 23x^3 + 60x^2 - 125x - 500) \div (x + 4)$
    $2x^3 + 15x^2 - 125$

Use synthetic division and the given factor to completely factor each polynomial function.

21. $y = x^3 + 3x^2 - 13x - 15; (x + 5)$
    $y = (x + 1)(x - 3)(x + 5)$
22. $y = x^3 - 3x^2 - 10x + 24; (x - 2)$
    $y = (x - 2)(x + 3)(x - 4)$
23. $y = x^3 + x^2 - 10x + 8; (x - 1)$
    $y = (x - 2)(x + 4)(x - 1)$
24. $y = x^3 + 4x^2 - 9x - 36; (x + 3)$
    $y = (x + 4)(x - 3)(x + 3)$
25. The expression $V(x) = x^3 - 13x + 12$ represents the volume of a rectangular safe in cubic feet. The length of the safe is $x + 4$. What linear expressions with integer coefficients could represent the other dimensions of the safe? Assume that the height is greater than the width.
    height $= (x - 1)$ ft; width $= (x - 3)$ ft

Use synthetic division and the Remainder Theorem to find $P(a)$.

26. $P(x) = 3x^3 - 4x^2 - 5x + 1; a = 2$
    -1
27. $P(x) = x^3 + 7x^2 + 12x - 3; a = -5$
    -13
28. $P(x) = x^3 + 6x^2 + 10x + 3; a = -3$
    0
29. $P(x) = 2x^4 - 9x^3 + 7x^2 - 5x + 11; a = 4$
    39

Name _____ Class _____ Date _____

Practice (continued) *Form G*

Dividing Polynomials

Divide.

30. $(6x^3 + 2x^2 - 11x + 12) \div (3x + 4)$
$2x^2 - 2x - 1$, R 16

31. $(x^4 + 2x^3 + x - 3) \div (x - 1)$
$x^3 + 3x^2 + 3x + 4$, R 1

32. $(2x^4 + 3x^3 - 4x^2 + x + 1) \div (2x - 1)$
$x^3 + 2x^2 - x$, R 1

33. $(x^5 - 1) \div (x - 1)$
$x^4 + x^3 + x^2 + x + 1$

34. $(x^4 - 3x^2 - 10) \div (x - 2)$
$x^3 + 2x^2 + x + 2$, R -6

35. $\left(3x^3 - 2x^2 + 2x + 1\right) \div \left(x + \frac{1}{3}\right)$
$3x^2 - 3x + 3$

36. The volume in cubic inches of a box can be expressed as the product of its three dimensions: $V(x) = x^3 - 16x^2 + 79x - 120$. The length is $x - 8$. Find linear expressions with integer coefficients for the other dimensions. Assume that the width is greater than the height. width: $x - 3$; height: $x - 5$

37. Writing What are the divisor, quotient, and remainder represented by the synthetic division below?

-5	1	0	-19	30
		-5	25	-30
	1	-5	6	0

divisor = $x + 5$; quotient = $x^2 - 5x + 6$; remainder = 0

38. Reasoning What does it mean if $P(-4)$ for the polynomial function $P(x) = x^3 + 11x^2 + 34x + 24$ equals zero?
It means that $(x + 4)$ is a factor of the polynomial.

39. Error Analysis Using synthetic division, you say that the quotient of $4x^3 - 3x^2 + 15$ divided by $x - 1$ is $4x^2 - 7x + 7$ R 8. Your friend says that the quotient is $4x^2 + x + 1$ R 16. Who is correct? What mistake was made?
Your friend is correct. You forgot to change the sign of the divisor from negative to positive.

40. What is $P(-2)$ for $P(x) = 3x^3 - 6x^2 + 2x - 12$?
-64

41. The expression $x^3 + 16x^2 + 68x + 80$ represents the volume of a flower box in cubic inches. The expression $x + 4$ represents the depth of the box. Assume that the length is greater than the height and that linear expressions with integer coefficients represent both.

a. What are the other dimensions of the flower box? length = $(x + 10)$ in.; height = $(x + 2)$ in.

b. If $x = 3$, what are the dimensions of the flower box? depth = 7 in.; length = 13 in.; height = 5 in.

Name _____ Class _____ Date _____

Practice

Form G

Theorems About Roots of Polynomial Equations

Use the Rational Root Theorem to list all possible rational roots for each equation. Then find any actual rational roots.

1. $x^3 + 5x^2 - 2x - 15 = 0$
   $\pm 1, \pm 3, \pm 5, \pm 15$; none
2. $36x^3 + 144x^2 - x - 4 = 0$
   $\pm 1, \pm 2, \pm 4, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{9}, \pm\frac{2}{9}, \pm\frac{4}{9}, \pm\frac{1}{12}, \pm\frac{1}{18}, \pm\frac{1}{36}$; $-4, -\frac{1}{6}, \frac{1}{6}$
3. $2x^3 + 5x^2 + 4x + 1 = 0$
   $\pm 1, \pm\frac{1}{2}$; $-1, -\frac{1}{2}$
4. $12x^4 + 14x^3 - 5x^2 - 14x - 4 = 0$
   $\pm 1, \pm 2, \pm 4, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{12}$; none
5. $5x^3 - 11x^2 + 7x - 1 = 0$
   $\pm 1, \pm\frac{1}{5}$; $\frac{1}{5}$, 1
6. $x^3 + 81x^2 - 49x - 49 = 0$
   $\pm 1, \pm 7, \pm 49$; none

A polynomial function $P(x)$ with rational coefficients has the given roots. Find two additional roots of $P(x) = 0$.

7. $2 + 3i$ and $\sqrt{7}$ $2 - 3i$, $-\sqrt{7}$
8. $3 - \sqrt{2}$ and $1 + \sqrt{3}$ $3 + \sqrt{2}$, $1 - \sqrt{3}$
9. $-4i$ and $6 - i$ $4i$, $6 + i$
10. $5 - \sqrt{6}$ and $-2 + \sqrt{10}$ $5 + \sqrt{6}$, $-2 - \sqrt{10}$
11. $\sqrt{5}$ and $-\sqrt{13}$ $-\sqrt{5}$ and $\sqrt{13}$
12. $1 - \sqrt{10}$ and $2 + \sqrt{2}$ $1 + \sqrt{10}$ and $2 - \sqrt{2}$

Write a polynomial function with rational coefficients so that $P(x) = 0$ has the given roots.

13. 4 and 6 $P(x) = x^2 - 10x + 24$
14. -5 and -1 $P(x) = x^2 + 6x + 5$
15. $3i$ and $\sqrt{6}$ $P(x) = x^4 + 3x^2 - 54$
16. $2 + i$ and $1 - \sqrt{5}$
    $P(x) = x^4 - 6x^3 + 9x^2 + 6x - 20$
17. -5 and $3i$ $P(x) = x^3 + 5x^2 + 9x + 45$
18. i and $5i$ $P(x) = x^4 + 26x^2 + 25$

What does Descartes' Rule of Signs say about the number of positive real roots and negative real roots for each polynomial function?

19. $P(x) = 3x^3 + x^2 - 8x - 12$ 1 positive real root; 2 or 0 negative real roots
20. $P(x) = 2x^4 - x^3 - 3x + 7$ 2 or 0 positive real roots; 0 negative real roots
21. $P(x) = 4x^5 - x^4 - x^3 + 6x^2 - 5$ 3 or 1 positive real roots; 2 or 0 negative real roots
22. $P(x) = x^3 + 4x^2 + x - 6$ 1 positive real root; 2 or 0 negative real roots

Name _____ Class _____ Date _____

Practice (continued) *Form G*

Theorems About Roots of Polynomial Equations

Find all rational roots for $P(x) = 0$.

23. $P(x) = x^3 - 5x^2 + 2x + 8$ **4, 2, −1**

24. $P(x) = x^3 + x^2 - 17x + 15$ **3, 1, −5**

25. $P(x) = 2x^3 + 13x^2 + 17x - 12$ **$-4, -3, \frac{1}{2}$**

26. $P(x) = x^3 - x^2 - 34x - 56$ **7, −2, −4**

27. $P(x) = x^3 - 18x + 27$ **3**

28. $P(x) = x^4 - 5x^2 + 4$ **−2, −1, 1, 2**

29. $P(x) = x^3 - 6x^2 + 13x - 10$ **2**

30. $P(x) = x^3 - 5x^2 + 4x + 10$ **−1**

31. $P(x) = x^3 - 5x^2 + 17x - 13$ **1**

32. $P(x) = x^3 + x + 10$ **−2**

33. $P(x) = x^3 - 5x^2 - x + 5$ **1, −1, 5**

34. $P(x) = x^3 - 12x + 16$ **−4, 2**

35. $P(x) = x^3 - 2x^2 - 5x + 6$ **−2, 1, 3**

36. $P(x) = x^3 - 8x^2 - 200$ **10**

37. $P(x) = x^3 + x^2 - 5x + 3$ **1, −3**

38. $P(x) = 4x^3 - 12x^2 - x + 3$ **$3, \frac{1}{2}, -\frac{1}{2}$**

39. $P(x) = x^3 + x^2 - 7x + 2$ **2**

40. $P(x) = 12x^3 + 31x^2 - 17x - 6$ **$-3, \frac{2}{3}, -\frac{1}{4}$**

Write a polynomial function $P(x)$ with rational coefficients so that $P(x) = 0$ has the given roots.

41. $\sqrt{3}, 2, -i$

$P(x) = x^5 - 2x^4 - 2x^3 + 4x^2 - 3x + 6$

42. $5, 2i$

$P(x) = x^3 - 5x^2 + 4x - 20$

43. $-1, 3 + i$

$P(x) = x^3 - 5x^2 + 4x + 10$

44. $-\sqrt{7}, i$

$P(x) = x^4 - 6x^2 - 7$

45. $-4, 4i$

$P(x) = x^3 + 4x^2 + 16x + 64$

46. $6, 3 - 2i$

$P(x) = x^3 - 12x^2 + 49x - 78$

47. Error Analysis A student claims that $2i$ is the only imaginary root of a polynomial equation that has real coefficients. Explain the student's mistake.
The student forgot the conjugate imaginary root $-2i$.

48. You are building a rectangular sandbox for a children's playground. The width of the sandbox is 4 times its height. The length of the sandbox is 8 ft more than 2 times its height. You have 40 ft^3 of sand available to fill this sandbox. What are the dimensions of the sandbox? **height = 1 ft, width = 4 ft, length = 10 ft**

49. Writing According to the Rational Root Theorem, what is the relationship between the polynomial equation $2x^4 - x^3 - 7x^2 + 5x + 3 = 0$ and rational roots of the form $\frac{p}{q}$, where $\frac{p}{q}$ is in simplest form?
p must be a factor of 3 and q must be a factor of 2.

Name _____ Class _____ Date _____

Practice

Form G

The Fundamental Theorem of Algebra

Without using a calculator, find all the complex roots of each equation.

1. $x^5 - 3x^4 - 8x^3 - 8x^2 - 9x - 5 = 0$
   $5, -1, -1, i, -i$
2. $x^3 - 2x^2 + 4x - 8 = 0$
   $-2, 2i, -2i$
3. $x^3 + x^2 - x + 2 = 0$
   $-2, \frac{1 + i\sqrt{3}}{2}, \frac{1 - i\sqrt{3}}{2}$
4. $x^4 - 2x^3 - x^2 - 4x - 6 = 0$
   $3, -1, i\sqrt{2}, -i\sqrt{2}$
5. $x^4 + 3x^3 - 21x^2 - 48x + 80 = 0$
   $4, -4, \frac{-3 + \sqrt{29}}{2}, \frac{-3 - \sqrt{29}}{2}$
6. $x^5 - 3x^4 + x^3 + x^2 + 4 = 0$
   $2, 2, -1, i, -i$

Find all the zeros of each function.

7. $y = 5x^3 - 5x$
   $-1, 0, 1$
8. $f(x) = x^3 - 16x$
   $-4, 0, 4$
9. $g(x) = 12x^3 - 2x^2 - 2x$
   $-\frac{1}{3}, 0, \frac{1}{2}$
10. $y = 6x^3 + x^2 - x$
    $-\frac{1}{2}, 0, \frac{1}{3}$
11. $f(x) = 5x^3 + 6x^2 + x$
    $-1, -\frac{1}{5}, 0$
12. $y = -4x^3 + 100x$
    $-5, 0, 5$

For each equation, state the number of complex roots, the possible number of real roots, and the possible rational roots.

13. $2x^2 + 5x + 3 = 0$
    2; 2 or 0; $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}$
14. $3x^2 + 11x - 10 = 0$
    2; 2 or 0; $\pm 1, \pm 2, \pm 5, \pm 10, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}$
15. $2x^4 - 18x^2 + 5 = 0$
    4; 4, 2 or 0; $\pm 1, \pm 5, \pm\frac{1}{2}, \pm\frac{5}{2}$
16. $4x^3 - 12x + 9 = 0$
    3; 3 or 1; $\pm 1, \pm 3, \pm 9, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{9}{4}$
17. $6x^5 - 28x + 15 = 0$ 5; 5, 3, or 1; $\pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{6}, \pm\frac{5}{6}$
18. $x^3 - x^2 - 2x + 7 = 0$
    3; 3, or 1; $\pm 1, \pm 7$
19. $x^3 - 6x^2 - 7x - 12 = 0$
    3; 3 *or* 1; $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$
20. $2x^4 + x^2 - x + 6 = 0$
    4; 4, 2, or 0; $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}$
21. $4x^5 - 5x^4 + x^3 - 2x^2 + 2x - 6 = 0$
    5; 5, 3, or 1; $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$
22. $7x^6 + 3x^4 - 9x^2 + 18 = 0$ 6; 6, 4, 2 or 0; $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm\frac{1}{7}, \pm\frac{2}{7}, \pm\frac{3}{7}, \pm\frac{6}{7}, \pm\frac{9}{7}, \pm\frac{18}{7}$
23. $5 + x + x^2 + x^3 + x^4 + x^5 = 0$
    5; 5, 3, or 1; $\pm 1, \pm 5$
24. $6 - x + 2x^3 - x^3 + x^4 - 8x^5 = 0$ 5; 5, 3, or 1; $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{1}{8}, \pm\frac{3}{8}$

Find the number of complex roots for each equation.

25. $x^8 - 5x^6 + x^4 + 2x - 16 = 0$ 8
26. $x^{10} - 100 = 0$ 10
27. $2x^4 + x^3 - 3x^2 + 4x - 2 = 0$ 4
28. $-4x^3 + x^2 - 3x + 10 = 0$ 3
29. $x^6 + 2x^5 + 3x^4 + 4x^3 + 5x^2 + 6x + 10 = 0$ 6
30. $-3x^5 + 4x^4 + 5x^2 - 15 = 0$ 5

Name _____ Class _____ Date _____

Practice (continued) *Form G*

The Fundamental Theorem of Algebra

Find all the zeros of each function.

31. $f(x) = x^3 - 9x^2 + 27x - 27$
3

32. $y = 2x^3 - 8x^2 + 18x - 72$
$4, -3i, 3i$

33. $y = x^3 - 10x - 12$
$-2, 1 \pm \sqrt{7}$

34. $y = x^3 - 4x^2 + 8$
$2, 1 \pm \sqrt{5}$

35. $f(x) = 2x^3 + x - 3$
$1, \frac{-1 \pm i\sqrt{5}}{2}$

36. $y = x^3 - 2x^2 - 11x + 12$
$-3, 1, 4$

37. $g(x) = x^3 + 4x^2 + 7x + 28$
$-4, -i\sqrt{7}, i\sqrt{7}$

38. $f(x) = x^3 + 3x^2 + 6x + 4$
$-1, -1 \pm i\sqrt{3}$

39. $g(x) = x^4 - 5x^2 - 36$
$-3, 3, -2i, 2i$

40. $y = x^4 - 7x^2 + 12$
$-2, 2, -\sqrt{3}, \sqrt{3}$

41. $y = 9x^4 + 5x^2 - 4$
$-\frac{2}{3}, \frac{2}{3}, -i, i$

42. $y = 4x^4 - 11x^2 - 3$
$-\sqrt{3}, \sqrt{3}, -\frac{1}{2}i, \frac{1}{2}i$

43. Error Analysis Your friend says that the equation $4x^7 - 3x^3 + 4x^2 - x + 2 = 0$ has 5 complex roots. You say that the equation has 7 complex roots. Who is correct? What mistake was made?
You are correct. Your friend may have counted the number of terms in the equation instead of using the Fundamental Theorem of Algebra.

44. A section of roller coaster can be modeled by the function $f(x) = x^5 - 5x^4 - 31x^3 + 113x^2 + 282x - 360$. A walkway bridge will be placed at one of the zeros. What are the possible locations for the walkway bridge? $-4, -3, 1, 5, 6$

45. Writing Using the Fundamental Theorem of Algebra, explain how $x^3 = 0$ has 3 roots and 3 linear factors. The Fundamental Theorem of Algebra says that the degree of the function is equal to the number of zeros. The degree of $x^3 = 0$ is 3. x^3 can be written as $x \cdot x \cdot x = 0$ or $(x - 0)(x - 0)(x - 0) = 0$. This shows that there are 3 linear factors and 3 zeros, all equal to 0.

46. How many complex roots does the equation $x^4 = 256$ have? What are they?
4 complex roots; $4, -4, 4i, -4i$

47. Reasoning Can a fifth-degree polynomial with rational coefficients have 4 real roots and 1 irrational root? Explain why or why not?
No; a polynomial with rational coefficients cannot have only 1 irrational root, it must have both conjugates. So, a fifth-degree polynomial can have 0, 2 or 4 irrational roots and 5, 3 or 1 real roots.

Name _____ Class _____ Date _____

Practice

Form G

The Binomial Theorem

Expand each binomial. See answers below.

1. $(x+2)^4$
2. $(a+2)^7$
3. $(x+y)^7$
4. $(d-2)^9$
5. $(2x-3)^8$
6. $(x-1)^9$
7. $(2x^2-2y^2)^6$
8. $(x^5+2y)^7$
9. $(n-3)^3$
10. $(2n+2)^4$
11. $(n-6)^5$
12. $(n-1)^6$
13. $(2a+2)^3$
14. $(x^2-y^2)^4$
15. $(2x+3y)^5$
16. $(2x^2+y^2)^6$
17. $(x^2-y^2)^3$
18. $(2b+c)^4$
19. $(3m-2n)^5$
20. $(x^3-y^4)^6$

Find the specified term of each binomial expansion.

21. third term of $(x+3)^{12}$ $594x^{10}$
22. second term of $(x+3)^9$ $27x^8$
23. twelfth term of $(2+x)^{11}$ x^{11}
24. third term of $(x-2)^{12}$ $264x^{10}$
25. eighth term of $(x-2y)^{15}$ $-823{,}680x^8y^7$
26. seventh term of $(x-2y)^6$ $64y^6$
27. fifth term of $(x^2+y^2)^{13}$ $715x^{18}y^8$
28. fourth term of $(x^2-2y)^{11}$ $-1320x^{16}y^3$
29. The term $126c^4d^5$ appears in the expansion of $(c+d)^n$. What is n? 9
30. The coefficient of the second term in the expansion of $(r+s)^n$ is 7. Find the value of n, and write the complete term. $n = 7$; $7r^6s$

State the number of terms in each expansion and give the first two terms.

31. $(d+e)^{12}$ 13; $d^{12} + 12d^{11}e$
32. $(x-y)^{15}$ 16; $x^{15} - 15x^{14}y$
33. $(2a+b)^5$ 6; $32a^5 + 80a^4b$
34. $(x-3y)^7$ 8; $x^7 - 21x^6y$
35. $(4-2x)^8$ 9; $65{,}536 - 262{,}144x$
36. $(x^2+y)^6$ 7; $x^{12} + 6x^{10}y$
37. The side of a number cube is $x+4$ units long. Write a binomial for the volume of the number cube. Use the Binomial Theorem to expand and rewrite the expression in standard form. $(x+4)^3 = x^3 + 12x^2 + 48x + 64$

1. $x^4 + 8x^3 + 24x^2 + 32x + 16$
2. $a^7 + 14a^6 + 84a^5 + 280a^4 + 560a^3 + 672a^2 + 448a + 128$
3. $x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$
4. $d^9 - 18d^8 + 144d^7 - 672d^6 + 2016d^5 - 4032d^4 + 5376d^3 - 4608d^2 + 2304d - 512$
5. $256x^8 - 3072x^7 + 16{,}128x^6 - 48{,}384x^5 + 90{,}720x^4 - 108{,}864x^3 + 81{,}648x^2 - 34{,}992x + 6561$
6. $x^9 - 9x^8 + 36x^7 - 84x^6 + 126x^5 - 126x^4 + 84x^3 - 36x^2 + 9x - 1$
7. $64x^{12} - 384x^{10}y^2 + 960x^8y^4 - 1280x^6y^6 + 960x^4y^8 - 384x^2y^{10} + 64y^{12}$
8. $x^{35} + 14x^{30}y + 84x^{25}y^2 + 280x^{20}y^3 + 560x^{15}y^4 + 672x^{10}y^5 + 448x^5y^6 + 128y^7$
9. $n^3 - 9n^2 + 27n - 27$
10. $16n^4 + 64n^3 + 96n^2 + 64n + 16$
11. $n^5 - 30n^4 + 360n^3 - 2160n^2 + 6480n - 7776$
12. $n^6 - 6n^5 + 15n^4 - 20n^3 + 15n^2 - 6n + 1$
13. $8a^3 + 24a^2 + 24a + 8$
14. $x^8 - 4x^6y^2 + 6x^4y^4 - 4x^2y^6 + y^8$
15. $32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5$
16. $64x^{12} + 192x^{10}y^2 + 240x^8y^4 + 160x^6y^6 + 60x^4y^8 + 12x^2y^{10} + y^{12}$
17. $x^6 - 3x^4y^2 + 3x^2y^4 - y^6$
18. $16b^4 + 32b^3c + 24b^2c^2 + 8bc^3 + c^4$
19. $243m^5 - 810m^4n + 1080m^3n^2 - 720m^2n^3 + 240mn^4 - 32n^5$
20. $x^{18} - 6x^{15}y^4 + 15x^{12}y^8 - 20x^9y^{12} + 15x^6y^{16} - 6x^3y^{20} + y^{24}$

Name _____ Class _____ Date _____

Practice (continued) *Form G*

The Binomial Theorem

Expand each binomial.

38. $(x + 1)^7$

$x^7 + 7x^6 + 21x^5 + 35x^4 + 35x^3 + 21x^2 + 7x + 1$

39. $(x + 4)^8$

$x^8 + 32x^7 + 448x^6 + 3584x^5 + 17{,}920x^4 + 57{,}344x^3 + 114{,}688x^2 + 131{,}072x + 65{,}536$

40. $(x - 3y)^6$

$x^6 - 18x^5y + 135x^4y^2 - 540x^3y^3 + 1215x^2y^4 - 1458xy^5 + 729y^6$

41. $(x + 2)^5$

$x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$

42. $(x^2 - y^2)^5$

$x^{10} - 5x^8y^2 + 10x^6y^4 - 10x^4y^6 + 5x^2y^8 - y^{10}$

43. $(3 + y)^5$

$y^5 + 15y^4 + 90y^3 + 270y^2 + 405y + 243$

44. $(x^2 + 3)^6$

$x^{12} + 18x^{10} + 135x^8 + 540x^6 + 1215x^4 + 1458x^2 + 729$

45. $(x - 5)^7$

$x^7 - 35x^6 + 525x^5 - 4375x^4 + 21{,}875x^3 - 65{,}625x^2 + 109{,}375x - 78{,}125$

46. $(x - 4y)^4$

$x^4 - 16x^3y + 96x^2y^2 - 256xy^3 + 256y^4$

47. Open-Ended Write a binomial in the form $(a + b)^n$ that has 3 as the coefficient of the first term. Answers may vary. Any binomial that has an a-term coefficient of $\sqrt[n]{3}$.

48. Use Pascal's Triangle to determine the binomial of the expanded expression $x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$. $(x + 1)^6$

49. Error Analysis Your friend expands the binomial $(x - 2)^6$ as $x^6 + 12x^5 + 30x^4 + 160x^3 + 240x^2 + 192x + 64$. What mistake did your friend make? What is the correct expansion? Your friend used $b = 2$ instead of $b = -2$, and forgot to square the 2; $x^6 - 12x^5 + 60x^4 - 160x^3 + 240x^2 - 192x + 64$

50. Reasoning Without writing any of the previous terms, how do you know that 2187 is the eighth term of the expansion of the binomial $(x + 3)^7$?

According to Pascal's Triangle, the eighth term of a binomial with $n = 7$ is equal to $1a^0b^7$. In this binomial $a = x$ and $b = 3$. $1x^03^7 = 2187$.

51. In the expansion of $(3x - y)^6$, one of the terms contains the factor y^4.

a. What is the exponent of $3x$ in this term? 2

b. What is the coefficient of this term? 135

52. You are shipping a cubic glass sculpture. Each side of the sculpture is x in. long. To adequately protect the sculpture, the shipping box must leave room for 5 in. of padding on either side in every dimension. Write and expand a binomial for the volume of the shipping box. $V = (x + 5)^3 = x^3 + 15x^2 + 75x + 125$ in.^{3}

Name _____ Class _____ Date _____

Practice

Form G

Polynomial Models in the Real World

Find a polynomial function whose graph passes through each set of points.

1. $(4, -1)$ and $(-3, 13)$
$y = -2x + 7$

2. $\left(1, -\frac{9}{2}\right)$ and $(6, -2)$
$y = \frac{1}{2}x - 5$

3. $(7, -5)$ and $(-1, 3)$
$y = -x + 2$

4. $(0, -3)$, $(-2, -7)$, and $(2, 9)$
$y = x^2 + 4x - 3$

5. $(-3, 15)$, $(1, 11)$, and $(0, 6)$
$y = 2x^2 + 3x + 6$

6. $(-2, -12)$, $(1, -6)$, and $(2, -24)$
$y = -5x^2 - 3x + 2$

7. $(4, -1)$, $(-2, -13)$, and $(1, 2)$
$y = -x^2 + 4x - 1$

8. $(0, 9)$, $(2, 21)$ $(-1, 0)$, and $(3, 36)$
$y = x^3 - 2x^2 + 6x + 9$

Find a polynomial function that best models each set of values.

9. Let $x =$ the number of years after 1985.

World Gold

Year	Production (millions of troy ounces)
1985	49.3
1990	70.2
1995	71.8
2000	82.6

SOURCES: *The World Almanac* and *World Gold*

$f(x) = 0.038x^3 - 0.956x^2 + 8.01x + 49.3$

10. Let $x =$ the number of years after 1970.

Life Expectancy

Year of Birth	Female (years)
1970	74.7
1980	77.4
1990	78.8
2000	79.7

SOURCE: U.S. Bureau of the Census

$f(x) = 0.00013x^3 - 0.0105x^2 + 0.3617x + 74.7$

11. Let $x =$ the number of years after 1985.

U.S. Energy

Year	Total Production ($\times 10^{15}$ Btu)
1985	64.9
1990	70.8
1995	71.0

SOURCE: Energy Information Administration

$f(x) = -0.114x^2 + 1.75x + 64.9$

12. Let $x =$ the number of years after 1980.

Social Security Benefits

Year	Monthly Average (dollars)
1980	321.10
1990	550.50
2000	844.60

SOURCE: *www.infoplease.com*

$f(x) = 0.3235x^2 + 19.705x + 321.1$

Find a cubic and a quartic model for each set of values. Then determine which model best represents the values.

13.

x	−2	−1	0	1	2
y	−7	−3	3	5	−3

$f(x) = -x^3 - 2x^2 + 5x + 3$;
$f(x) = 0x^4 - x^3 - 2x^2 + 5x + 3$;
the cubic best represent the values.

14.

x	−2	−1	0	1	2
y	2	−6	2	8	42

$f(x) = x^3 + 5.86x^2 + 6x - 2.11$;
$f(x) = 2x^4 + x^3 - 3x^2 + 6x + 2$;
the quartic model best represents the values.

Name _____ Class _____ Date _____

Practice (continued) *Form G*

Polynomial Models in the Real World

Use your models from Exercises 9–12 to make predictions.

15. Estimate world gold production for 2010, 2020, and 2025.
245.8 troy oz., 787.8 troy oz., 1272.1 troy oz.

16. Estimate the life expectancy for women born in 1986, 1992, and 2005.
78.3 years, 79.0 years, 80.1 years

17. Estimate the U.S. energy production for 2002, 2005, and 2010.
61.7×10^{15} Btu, 54.3×10^{15} Btu, 37.4×10^{15} Btu

18. Estimate the average monthly Social Security benefits for 1970, 1996, and 1999.
$156.40, $719.20, $812.28

19. Find a cubic function to model the data below. (*Hint*: Use x to represent the gestation period.) Then use the function to estimate the longevity of an animal with a gestation period of 151 days. **$0.0000006x^3 - 0.0005101x^2 + 0.1270416x + 2.0612682$; about 12 yr**

Gestation and Longevity of Certain Animals

Animal	Rat	Squirrel	Pig	Cow	Elephant
Gestation (in days)	21	44	115	280	624
Longevity (in years)	3	9	10	12	40

SOURCE: *www.infoplease.com*

20. Error Analysis Your teacher gives the class the table at the right and asks you to find a polynomial model for the data set. Then he asks the class to estimate the percent of U.S. foreign-born population in 1920. Your friend uses $x = -10$ and estimates the percent as 16.1. What did your friend do wrong? What is the correct estimate?
Your friend should have used $x = 10$ because 1920 is 10 years after the data set began, not 10 years before. The correct estimate is 13.1%.

U.S. Population

Year	Foreign-Born (percent)
1910	14.7
1930	11.6
1950	6.9
1970	4.7
1990	8.0
2000	10.4
2004	11.7

SOURCE: Bureau of the Census

21. Reasoning Using the data set from Exercise 12 and the model you determined, find the average monthly Social Security benefits for the year 2050. Do you have much confidence in this prediction? Explain.
$3285.60; no, because the data point is so far from the data set used to create the model. There is no way to predict what changes may occur between 2000 and 2050.

22. Find a cubic model for the following set of values: $(0, -4)$, $(-1, -6)$, $(5, -264)$, and $(2, -18)$. Using the regression coefficient, determine whether the model is a good fit.
$y = -2x^3 - x^2 + 3x - 4$; this is a good fit because $R^2 = 1$.

Name _____ Class _____ Date _____

Practice

Form G

Transforming Polynomial Functions

Determine the cubic function that is obtained from the parent function $y = x^3$ after each sequence of transformations.

1. a reflection in the x-axis;
   a vertical translation 3 units down;
   and a horizontal translation 2 units right
   $y = -(x - 2)^3 - 3$

2. a vertical stretch by a factor of 4;
   a reflection in the x-axis;
   and a horizontal translation $\frac{1}{2}$ unit left
   $y = -4\left(x + \frac{1}{2}\right)^3$

3. a vertical stretch by a factor of $\frac{1}{3}$;
   a reflection in the y-axis;
   and a vertical translation 6 units up
   $y = \frac{1}{3}(-x)^3 + 6$

4. a vertical stretch by a factor of 3;
   a reflection in the x-axis;
   a vertical translation 2 units down;
   and a horizontal translation 2 units left
   $y = -3(x + 2)^3 - 2$

Find all the real zeros of each function.

5. $y = 2(x + 1)^3 - 3$ $x = \sqrt[3]{\frac{3}{2}} - 1$

6. $y = -3(x - 2)^3 + 24$ $x = 4$

7. $y = -\frac{1}{2}(x + 4)^3 - 1$ $x = \sqrt[3]{-2} - 4$

8. $y = 8(-x - 2)^3 + 5$ $x = \frac{\sqrt[3]{5}}{2} - 2$

9. $y = -(x + 5)^3 + 1$ $x = -4$

10. $y = 4(x - 6)^3 - 2$ $x = \frac{1}{\sqrt[3]{2}} + 6$

Find a quartic function with the given x-values as its only real zeros.

11. $x = 2$ and $x = 8$
    Answers may vary. Sample:
    $y = (x - 5)^4 - 81$

12. $x = 3$ and $x = -1$
    Answers may vary. Sample:
    $y = (x - 1)^4 - 16$

13. $x = 1$ and $x = 3$
    Answers may vary. Sample:
    $y = (x - 2)^4 - 1$

14. $x = -2$ and $x = 6$
    Answers may vary. Sample:
    $y = (x - 2)^4 - 256$

15. $x = 5$ and $x = -2$
    Answers may vary. Sample:
    $y = x^4 - 3x^3 - 9x^2 - 3x - 10$

16. $x = -1$ and $x = 2$
    Answers may vary. Sample:
    $y = x^4 - x^3 - x^2 - x - 2$

17. $x = -3$ and $x = -5$
    Answers may vary. Sample:
    $y = x^4 + 8x^3 + 16x^2 + 8x + 15$

18. $x = -4$ and $x = 4$
    Answers may vary. Sample:
    $y = x^4 - 15x^2 - 16$

19. **Physics** If you stretch a spring to 5 ft, it has 310 ft-lb of potential energy (PE). Potential energy varies directly as the square of the stretched length (l). The potential energy can be represented by the formula $PE = \frac{1}{2}kl^2$, where k is the spring constant.
    a. What is the value of the spring constant for this spring? 24.8
    b. How many ft-lbs of PE would an 8 ft length of spring have? 793.6 ft-lb

Name _____ Class _____ Date _____

Practice (continued) *Form G*

Transforming Polynomial Functions

Determine whether each function can be obtained from the parent function $y = x^n$, using basic transformations. If so, describe the sequence of transformations.

20. $y = 2(x - 3)^3 + 4$ yes; vertical stretch by a factor of 2, horizontal translation 3 units right, vertical translation 4 units up

21. $y = x^4 + x - 3$ no

22. $y = -\frac{1}{3}x^2$ yes; vertical stretch by a factor of $\frac{1}{3}$, reflection across *x*-axis

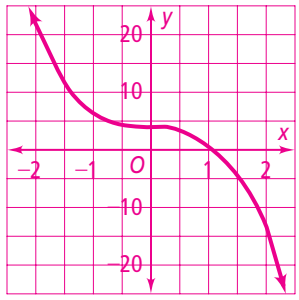
23. $y = (-x + 5)^3$ yes; reflection across *y*-axis, horizontal translation 5 units right

24. $y = \frac{2}{x^3}$ yes; vertical stretch by a factor of 2

25. $y = 4(x)^4 - 12$ yes; vertical stretch by a factor of 4, vertical translation 12 units down

26. Graph the parent function $y = x^3$ after it has been transformed by the following changes.

- vertical stretch by a factor of $2\frac{1}{4}$
- reflection across the *x*-axis
- vertical translation 4 units up



27. Error Analysis Your friend set up a problem to find a quartic function with the only real zeros of $x = -4$ and $x = 1$. She wrote down $y = (x + 4)(x - 1)(x^2 - 1)$. Will she get a correct quartic function? Why or why not?
No; she used a quadratic that has additional real zeros; she should have used $x^2 + 1$ as her quadratic.

28. Open-Ended Transform the parent function $y = x^3$ by vertical stretch, reflection across the *x*-axis, horizontal translation, and vertical translation.
Any cubic polynomial in the form of $y = a(x - h)^3 + k$ where $a < 0$ and h and $k \neq 0$.

29. You are swinging a bucket in a circle at a velocity of 7.8 ft/s. The radius of the circle you are making is 1.25 ft. The acceleration is equal to one over the radius multiplied by the velocity squared.

a. What is the acceleration of the bucket? about 48.7 ft/s^2

b. What is the velocity if the acceleration is 25 ft/sec^2? about 5.6 ft/s